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Brane inflation, solitons and cosmological solutions: I

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Abstract: In this paper we study various cosmological solutions for a D3/D7 system directly from M-theory with fluxes and M2-branes. In M-theory, these solutions exist only if we incorporate higher derivative corrections from the curvatures as well as G-fluxes. We take these corrections into account and study a number of toy cosmologies, including one with a novel background for the D3/D7 system whose supergravity solution can be completely determined. This new background preserves all the good properties of the original model and opens up avenues to investigate cosmological effects from wrapped branes and brane-antibrane annihilation, to name a few. We also discuss in some detail semilocal defects with higher global symmetries, for example exceptional ones, that could occur in a slightly different regime of our D3/D7 model. We show that the D3/D7 system does have the required ingredients to realise these configurations as non-topological solitons of the theory. These constructions also allow us to give a physical meaning to the existence of certain underlying homogeneous quaternionic Kähler manifolds.

Keywords: F-Theory, Supergravity Models, D-branes, M-Theory.

Contents

1. Introduction

One of the most intriguing results of the recent spectacular advances in observational cosmology is the strong evidence for at least two periods of accelerated expansion in the history of our universe. The second, presently ongoing, of these two periods is commonly attributed to a tiny positive cosmological constant, or some similar form of *dark energy*. The other, much earlier, phase of accelerated expansion refers to some period well before Big Bang Nucleosynthesis and is commonly known as *inflation* [1−3]. Inflation is usually modelled by a scalar field, the inflaton, which is slowly rolling down a sufficiently flat potential that dominates the energy density during that epoch. A phase of early universe inflation not only solves several longstanding cosmological problems such as the horizon, flatness or potential monopole problems, but also provides a natural mechanism for the generation of density perturbations. The resulting predictions for the anisotropies of the cosmic microwave background (CMB) have been nicely confirmed by recent experiments such as the Wilkinson microwave anisotropy probe (WMAP) [4].

In view of this observational evidence, it is now tempting to include inflation in the list of desirable properties any fundamental theory of our world should be able to explain or at least reproduce. String theory aspires to be such a fundamental theory, and should therefore be confronted with this evidence. Embedding inflationary models in string theory touches upon several problems:

(i) Time-dependence: describing cosmological spacetimes in string theory in general means dealing with time-dependent backgrounds, which are necessarily non-supersymmetric and hence, harder to control.

(ii) Acceleration: reproducing longlasting accelerating spacetimes from string compactifications is particularly non-trivial.

(iii) Identification of the inflaton: the inflaton should be identified with a (possibly effective) scalar field in string theory. The moduli in string compactifications naturally provide scalar fields, but a priori, it is not clear, which, if any, of these moduli fields should be identified with the inflaton.

(iv) Moduli stabilization: if one direction of the moduli space is identified with the inflaton, the scalar potential along this direction should be sufficiently flat, whereas all the other directions should be stabilized with sufficiently large masses in order not to interfere with the slow-roll of the inflaton. This is in general an issue of fine-tuning.

(v) End of inflation: if the inflaton is a modulus, how can the end of inflation and reheating be understood geometrically? Are there any unwanted relics after inflation, and if so, what is their stringy origin?

As for problem (ii), no-go theorems against accelerating spacetimes have been proven for compactifications of *classical* 10D or 11D supergravity on *smooth*, *compact* and *time-independent* internal spaces [5]. By relaxing any of the highlighted assumptions, this no-go theorem might possibly be circumvented. For example, allowing for a time-dependent hyperbolic internal space [6, 7] or a time-dependent flux compactification [8 − 12, 7] opens up the possibility of having transient periods of inflation. These solutions are in fact related to S-branes [13]. Such transient periods of accelerated expansion might be marginally compatible with the present acceleration [14], but they are far too short to describe the required 50 - 60 efolding of early universe inflation [9, 8, 15] (see also [16] for some related recent work).

From the effective 4D point of view, this is related to the fact that the volume modulus generically enters the scalar potentials of string compactifications in terms of an exponential factor with a coefficient that makes the potential too steep for slow-roll inflation. There are essentially two ways around this problem: Either there are additional, non-classical contributions to the scalar potential that could lead to a sufficiently flat regime [17], or the volume modulus should be fixed during inflation altogether. In the latter case, the inflaton has to be another scalar field.

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This latter possibility is entertained in "brane inflation" models [18], where the inflaton is identified with an open string modulus that describes the relative position or orientation of a D-brane with respect to another (system of) D-brane(s). A necessary ingredient of these models is that the brane configuration breaks supersymmetry so that the branes attract and move towards one another. In brane/anti-brane systems [19], supersymmetry is not just mildly broken, and the interaction potential would be too steep if the branes were too close to one another. It was pointed out in [20, 21] that the finite size of a compact space puts limits on the possible interbrane distance, and hence, the possible flatness of the brane/anti-brane potential that are in general incompatible with slow roll inflation. This problem has been solved by putting the anti-brane in a warped throat region [21, 22].

Another way to get reasonably flat brane interaction potentials is to consider brane configurations that break supersymmetry more gently than brane/anti-brane pairs. Examples for such configurations include intersecting branes with a small relative angle [23, 24] or branes with small supersymmetry breaking worldvolume fluxes. The prime example for the latter possibility is provided by the D3/D7 system with non-primitive fluxes on the worldvolume of the D7 brane [25, 24]. Classically the interaction potential between the D3 and the D7 brane is completely flat, but one-loop corrections due to the broken supersymmetry induce a logarithmic slope. An explicit realization of this model in a concrete string compactification was given in [25], where the compact space was chosen to be $K3 \times T^2/\mathbb{Z}_2$. This background was chosen because it is non-trivial, but still well under theoretical control with a well understood F-theory description. The effective low energy theory in four dimensions is given by a hybrid D-term [26] (or, more generally, "P-term" [27]) inflation model.[1]

As we will frequently refer to various aspects of this hybrid inflation model, let us briefly recall its basic features for later reference. There are three different pictures we will sometimes use for this model. The first is the original picture of hybrid inflation [29], which can be viewed as the "core" of these hybrid inflation models in the sense that it only uses the minimal field content and does not refer to any underlying more fundamental theory. The second picture is the supergravity P-term action of [27], where hybrid inflation is related to $\mathcal{N}=2$ supergravity. The third picture derives from the stringy analysis of the D3/D7 inflationary model [25, 24, 30]. The original picture of Linde [29] is based on the following potential written in terms of two scalar fields, ϕ and ψ:

$$V(\phi,\psi) = \frac{1}{4\lambda}(M^2 - \lambda\psi^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\psi^2\,, \tag{1.1}$$

where M, m, λ and g have been defined in [29]. In this model, ϕ is the inflaton which is slowly rolling down its potential, while ψ is temporarily trapped in its false vacuum with $\psi = 0$. When ϕ reaches a critical value, ψ becomes tachyonic, and the "waterfall" stage begins, where ψ quickly relaxes to its true vacuum state, thereby ending inflation.

[1] In a different context, low energy effective actions of D3/D7 systems with fluxes were recently studied in [28].

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This construction can be embedded in the slightly less minimal P-term model developed by Kallosh and Linde [27]. It is basically an $\mathcal{N}=2$ supergravity potential with FI terms,[2] and has the following potential:

$$V(\Phi_\pm, S) = 2g_1^2\left(|S|^2(|\Phi_+|^2+|\Phi_-|^2)+\left|\Phi_+\Phi_- - \frac{\zeta_+}{2}\right|^2\right)+\frac{g_1^2}{2}(|\Phi_+|^2-|\Phi_-|^2-\zeta_3)^2\,, \quad (1.2)$$

where S is the scalar in an $\mathcal{N}=2$ vector multiplet and (Φ_+,Φ_-) form the scalars of an $\mathcal{N}=2$ hypermultiplet. In terms of $\mathcal{N}=1$ language Φ_+ and Φ_- form the scalars of two chiral multiplets. The coupling constant is denoted by g_1 and $(\zeta_\pm,\zeta_3)$ form a triplet of FI terms [27].

The third and final picture which we will use derives from the D3/D7 inflationary model [25]. The potential is written in terms of the BI and CS terms of the D3 as well as the D7 branes [30] and is given by

$$V(S,\chi) = c_o\int_{K3}\mathcal{F}^-\wedge *\mathcal{F}^- + 2(g_3^2+\tilde{g}_3^2)|S|^2|\chi|^2+\frac{g_3^2+\tilde{g}_3^2}{2}(\chi^\dagger\sigma^A\chi)^2\,, \quad (1.3)$$

where σ^A are the Pauli matrices, g_3 denotes the coupling of the D3 brane, and $\tilde{g}_3$ is the effective four-dimensional coupling of the D7 branes. We have also normalised the action in such a way that the kinetic term of S is simply $|\partial S|^2$. The coefficient c_o is related to the volume of the internal space and the seven-brane coupling constant. In writing this potential, we have ignored the degrees of freedom that determine the center-of-mass motion of the system.

A detailed comparison between (1.2) and (1.3) can be found in [30], so we will be brief here. If we take the FI terms in (1.2) as $(\zeta_\pm,\zeta_3)=(0,\zeta_3)$, then ζ_3 can be identified with $\mathcal{F}^-$. The precise identification is

$$c_o\int_{K3}\mathcal{F}^-\wedge *\mathcal{F}^- \to g_1^2\zeta_3^2\,,\qquad S\to S\,,\qquad \chi_1\to\Phi_+\,,\qquad \chi_2\to\Phi_-\,,\qquad g_3^2+\tilde{g}_3^2\to g_1^2\,. \quad (1.4)$$

From the above identifications, it is now easy to write the gauge fluxes $\mathcal{F}^-$ in terms of the variables of [27]. We can use any of the harmonic $(1,1)$ forms Ω_i $(i=1\ldots h_{11})$ on the $K3$ manifold to represent the gauge flux as

$$\mathcal{F}^- = \sum_{i=1}^{h_{11}} c_i\,\Omega_i\,,\qquad \text{such that}\int_{K3}\Omega_i\wedge *\Omega_j=\delta_{ij}\,. \quad (1.5)$$

Written in terms of the harmonic forms one can easily show that the c_i form a surface in an h_{11} dimensional space. The equation for the surface can be written in terms of the variables of [27] and is explicitly given as

$$c_1^2+c_2^2+\cdots+c_{h_{11}}^2 = c_o^{-1}\,g_1^2\,\zeta_3^2\,. \quad (1.6)$$

[2]Strictly speaking, the $\mathcal{N}=2$ supersymmetry is broken to $\mathcal{N}=1$ by the coupling to supergravity due to the presence of the Fayet-Iliopoulos terms. Only in the rigid limit of global supersymmetry, the full $\mathcal{N}=2$ supersymmetry is restored.

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Taking the c_i to be real, the above equation basically defines a sphere $S^{h_{11}-1}$, with radius $r \equiv c_o^{-\frac{1}{2}} g_1 \, \zeta_3$. Therefore, one simple solution for the gauge flux would be to take the radius of $S^{h_{11}-1}$ as the antiselfdual flux, i.e $\mathcal{F}^- = r$. With this choice, the two models (1.2) and (1.3) can be easily identified.

To compare (1.3) (or (1.2)) with (1.1), one should first identify which fields in (1.2) become tachyonic. The mass of the hypers is given by $m^2_{\text{hyper}} = g_1^2|S|^2 \pm g_1\zeta_3$, which becomes tachyonic when $|S| < |S|_c$, where $|S|_c = \sqrt{\frac{\zeta_3}{g_1}},$. It is now easy to make the comparison with (1.1). The mass of the scalar field ψ is given by $m^2_\psi = -M^2 + g^2\phi^2$. This becomes tachyonic when $\phi < \phi_c$, with $\phi_c = \frac{M}{g}$. Therefore, one sees that $|S|$ is related to ϕ while $|\chi|$ is related to ψ. The coupling constant and the FI terms of the D3/D7 system are then related to g and M of [29] respectively. This concludes our little review of hybrid inflation in the context of the D3/D7 inflationary model.[3] As we have seen, a certain supergravity variant of the hybrid inflation potential arises naturally in the D3/D7 system with non-selfdual fluxes on the D7-brane worldvolume. The nice feature of this model is that the dominant contribution to the vacuum energy derives from a D-term in the supergravity potential. This circumvents the well-known eta-problem of supergravity models of inflation that are based on F-terms. In F-term inflation, the appearance of the Kähler potential in an exponential factor of any F-term potential in supergravity generically leads to the problematic relation, $\eta \sim 1$, where η is the second slow-roll parameter, $\eta = M_P^2 V''/V$ [33, 34]. Unfortunately, the currently known methods for volume stabilization are based on F-term potentials from non-perturbative superpotentials [35]. Thus, in general, even if the primary inflationary potential is a D-term potential, the supergravity eta-problem reenters through the back door via volume stabilization. This can be avoided if the Kähler potential posesses an inflaton shift symmetry [36, 37] (see also [38]). Interestingly, it was shown in [37] that, at tree level, the D3/D7 model on $K3 \times T^2/\mathbb{Z}_2$ precisely has such an inflaton shift symmetry, which is related to the continuous isometries of the two-torus. This inflaton shift symmetry is broken by one-loop corrections, already at the field theory level, and these one-loop corrections are precisely needed to slightly lift the classically flat direction of the inflaton potential. However, as pointed out in [39], the stringy one-loop corrections computed in [40] also violate the shift symmetry in a manner that the eta problem generically reappears. On the other hand, it can in general be removed by a modest ($\sim 10^{-2}$) fine-tuning of the fluxes or other parameters [39]. The necessity of such a modest fine tuning (which might not be such a big problem after all, as the models with sufficient efolding of inflation are "rewarded" by a much larger amounts of the spacetime they fill out) seems to be a generic feature of many stringy models of inflation, whether they are based on closed or open string moduli [39]. In retrospect, this does not seem very

[3] We should briefly mention yet another model that also shows hybrid inflation. This is the model studied by Guth-Randall-Soljacic [31], which in some regions, show properties identical to the original hybrid inflationary model [29]. The potential for inflationary behavior in this model is given by $V(\phi,\psi) = M^4 \cos^2 \frac{\psi}{f} + \frac{1}{2}m^2\phi^2 + g_4^2\phi^2\psi^2$, where the coefficients are explained in [31]. A detailed comparison between this model and (1.1) has already appeared in [32] so the readers can pick up details from there. We should simply point out that in the region $\psi < f$ this model and (1.1) are more or less identical.

surprising, as the very idea of all but one modulus having large masses while the remaining one gives rise to a comparatively flat potential does not, intuitively, look like a very generic situation. It is nevertheless interesting to explore how deeply hidden such fine-tunings can be in different models, and the D3/D7 model is certainly one of the models in which this is least obvious.[4]

Leaving this issue of modest and probably not too worrysome fine-tuning aside for a moment, it is worth emphasizing that the brane inflation models have generally been studied in a way that is technically different from that of the models of transient accelerating cosmologies in [6, 8 − 12]. More precisely, whereas in those papers the whole system is studied from a genuinely 10D or 11D point of view, the brane inflation models recover inflating four-dimensional spacetimes from an effective four-dimensional action that derives from an, a priori static, string compactification in which quantum effects and non-perturbative objects such as branes also play a role. This usage of effective field theories is sometimes criticized [42], among other things, because the 4D effective actions are often derived from compactifications on static backgrounds, which are, in general, not related by small perturbations to the time-dependent cosmological solutions they are ultimately used for. This would not be an issue if one were dealing with a *consistent truncation* instead of a mere low-energy effective action.[5] It should be emphasized, however, that so far there hasn't been any clear evidence that the usage of 4D effective field theory is inappropriate in this context.

Another issue of many brane inflationary models is that they tend to produce cosmic strings or other defects at the end of inflation. The energy density contributed by such defects should of course be small enough so as not to upset the successful predictions of inflation such as the CMB power spectrum. On the other hand, these defects can often be identified with inherently stringy objects such as the fundamental string itself or various types of (possibly wrapped) D-branes. This opens up an exciting possibility that, provided the abundance and energy density of such objects turn out to be in the right ballpark, one could study genuinely stringy objects by direct observations "at the sky". For more details on the recent work on this subject, see, e.g., [46, 47].

In this paper, we turn our attention to some of the aspects of D3/D7 brane inflation that have to do with the higher-dimensional and stringy nature of this scenario. More precisely, we make a first step towards the uplift of the full time-dependent 4D cosmology of the D3/D7 scenario to the full higher-dimensional spacetime picture. The purpose of this

[4]Since the time we submitted our paper, an important update regarding the moduli stabilization in the $K3 \times T^2/\mathbb{Z}_2$ model appeared in [41], where moduli stabilization due to euclidean D3-brane instantons instead of gaugino condensation is discussed. The results of [41] should have interesting applications for the D3/D7 inflationary model in that background.

[5]For a consistent truncation, any solution of the lower-dimensional theory can, by definition, be uplifted to a valid solution of the higher-dimensional theory, but for a mere effective low-energy theory, this need not necesarily be true. A particularly nice illustration of the power of consistent truncations is provided by 5D, $\mathcal{N} = 8$ gauged supergravity [43, 44], which is believed to be a consistent truncation of the compactification of IIB supergravity on the five-sphere to the lowest lying Kaluza-Klein modes. Because of this, domain wall solutions of this five-dimensional theory are guaranteed to have a well-defined up-lift to ten dimensions [45] and can thus reliably encode information on renormalization group flows of the deformed 4D, $\mathcal{N} = 4$ super Yang-Mills theory.

exercise is to better understand how precisely the no-go theorems of [5] are circumvented, and which effects are the most dominant. A successful uplift would also help to get some better insight into the validity of 4D effective field theory actions in this context. Doing this directly, however, would obviously require dealing with the seven-branes, three-branes, different types of fluxes and non-perturbative effects directly in IIB string theory or F-theory. This is clearly a highly non-trivial problem, and we will therefore use the slightly simpler uplift to the eleven-dimensional M-theory.[6] This has the advantage that the seven-branes (including the the non-perturbative ones that might result from orientifold planes) dissolve into background geometry. More precisely, they enter as singularities of the torus fibration of the fourfold, which as a whole, however, becomes a smooth eight-dimensional manifold. Furthermore, in the M-theory picture one has to deal with only one type of flux, namely that of the four-form field strength. Still, a complete uplift of the D3/D7-scenario represents a challenging problem, and in this first part, we limit ourselves to setting up and identifying all the necessary ingredients as well as to the study of various toy cosmologies. A more refined treatment is postponed to a later publication [49]. One of the main results in the present paper is that accelerating spacetimes in this scenario seem to require the inclusion of the higher derivative terms of the M-theory action and that the assumption of a single time-dependent warp factor for the internal manifold in the M-theory picture is probably too simplistic.

One of the drawbacks of the manifold $K3 \times T^2/\mathbb{Z}_2$ is that its metric is not explicitly known, except in the orbifold limit of the $K3$. We will therefore also construct a non-trivial background whose metric can be explicitly written down. This background is a compactified version of a non-Calabi-Yau deformed conifold and has a well-defined F-theory description in terms of a Calabi-Yau fourfold.[7] This background also has a non-trivial three-cycle around which D3-branes can wrap. Such three-cycles are absent in the $K3 \times T^2/\mathbb{Z}_2$ background and allow for certain charged massive black hole configurations. The excitation of such black holes during or after inflation are generally expected to be negligible, and in general, this is confirmed by our considerations. This, however, is not necessarily true for cosmic strings which, in the generic D3/D7 model, can easily ruin the otherwise successful predictions of inflation. In [30], a mechanism has been proposed by which the production rate of such cosmic strings can be sufficiently suppressed without going to unnatural regions of the parameter space. In this mechanism, one replaces the single D7-brane of the D3/D7 scenario by a stack of two or more coincident D7-branes. This results in an additional (approximately) global symmetry, and the resulting cosmic strings are the so-called semilocal strings [51, 52] instead of the conventional Abrikosov-Nielsen-Olesen (ANO) strings [53], which have a much lower formation rate [54, 51, 55]. In this paper, we take the non-perturbative effects of F-theory seriously and consider the possibility that the non-perturbative exceptional symmetries of certain F-theory configurations can

[6]Recently cosmological models directly from M-theory were also studied in [48]. However the emphasis of these models were to get four-dimensional cosmology *without* any brane inflation from M-theory. Thus our analysis is completely different from [48].

[7]Notice that this is different from the Klebanov-Strassler model [50] in the sense that it is compact, non-supersymmetric, non-Kähler and, as we will show later, has seven branes.

give rise to more general semilocal strings based on non-unitary symmetry groups, such as, e.g., symplectic or exceptional groups. The possibility of such semilocal strings was entertained in [56], but no explicit field theory realization of such strings is known to exist so far. Here we will provide a construction of these defects as non-topological solitons on the D3 brane.

1.1 Organization of the paper

The organization of this paper is as follows: In section 2 we set up the basic picture to study cosmological solutions for the D3/D7 system. We do not assume any particular embedding, but, a priori, allow all possible backgrounds for the branes to propagate. Our strategy here will be to study the system directly from M-theory, where the D3 branes will become M2 branes and the fluxes will become G-fluxes. In section 2.1 the background equations of motion, given by the Einstein and flux equations, are spelled out completely. We provide some of the possible quantum terms at this stage. Later on we will specify more quantum terms as the given set of terms would not suffice. In that section we also specify the criteria to obtain accelerating cosmologies, such as de Sitter spacetimes.

The full analysis of the background equations is worked out in section 2.2. We start with a very generic ansatz for the background metric that would allow three warp factors: one for the 2+1 dimensional spacetime and two for the internal manifold. The internal manifold for our case will be a complex fourfold which is a torus fibration over a six dimensional base, with possibly different warp factors along the fiber and along the six dimensional base. We will discuss how the curvature tensors etc should be written with three warp factors. Because of the large number of components of the Einstein tensor, the equations naturally get very involved. To simplify the subsequent analysis we consider only the two warp factor cases here in detail and discuss the three warp factor case only briefly. Taking two warp factors simplifies things a little bit but is still sufficiently interesting. In section 2.2 many of the interesting effects that will form the basis of our subsequent analyis are pointed out: supersymmetry breaking by non-primitive fluxes and the resulting motion of the M2 branes, the constraint coming from anomaly equations and the corresponding warp factor dependence, an analysis of every component of the Einstein tensor, as well as the effects coming from membrane and quantum terms and some surprising simplifications that would happen if some special relations are considered for the warp factors.

We then give many toy examples in sec 3. In most of these examples, we take some limit of the background fluxes, the size of the fourfold and the membrane velocities so that we can have some control over the quantum corrections. In this section we also point out the relevance of higher derivative terms that are in *addition* to the quantum terms that we took into account in section 2. These higher derivative terms (for G-fluxes and curvatures) are necessary to overcome the no-go theorem for warped compactification on a compact manifold [5].

In section 3.1 we take very small internal G-fluxes on an essentially non-compact fourfold with a slowly moving membrane. This way, one can study a background by ignoring the quantum corrections (without violating the no-go theorem).

In section 3.2 we consider a little more elaborate analysis by taking arbitrary fluxes. We find solutions *only* when higher derivative terms in G-fluxes (and possibly the curvatures) are incorporated here. Our solutions are thus valid to some order in quantum corrections. When the G-fluxes are not very large, and the fourfold is sufficiently big, the quantum effects may be kept under control without destabilising the solution.

In all the earlier examples above, we give cosmological solutions for which the fourfold have time independent warp factors. In section 3.3 we give yet another cosmological solution in type-IIB theory with two warp factors, but this time take both of them as time dependent. We again provide the complete background (up to the possible approximations that we consider), which turns out to be a radiation-dominated cosmology in four dimensions. In this context, we also briefly discuss some interesting questions on the interpretation and validity of cosmological solutions to supergravity in the presence of higher derivative terms.

In all the previous examples, it was difficult to generate an accelerating cosmology. In section 3.4, we come back to the three warp factor case. With three warp factors, a de-Sitter type cosmological solution in type-IIB might, in principle, be possible. We show that the higher derivative terms in G-fluxes may not be a big problem for this case.

In all the above cases, the internal fourfold was kept completely arbitrary. In section 3.5 we consider a new background that is different from the originally considered $K3 \times \mathbb{P}^1$ background of [25]. Just as the $K3 \times \mathbb{P}^1$ background, the new background allows D3 branes and also D7 branes along with G-fluxes. There is also a detailed F-theory description of this manifold in terms of a Weierstrass equation that we specify completely. To derive this background, we use the trick of *geometric transition* that was developed for the supersymmetric case in [57, 58]. The background that we eventually get is a non-supersymmetric and non-Kähler compact manifold that has some resemblance to the deformed conifold in some regime. Globally the manifold is quite different from a Calabi-Yau deformed conifold, and allows three cycles as well as four-cycles. We show that the metric of the manifold can be completely determined and present the precise *unwarped* metric. This should be contrasted with the $K3 \times \mathbb{P}^1$ background whose metric can only be determined when the K3 is at its orbifold point.

In section 3.6 and 3.7 we use some of the geometric properties of this manifold to study consequences on four dimensional cosmology from branes wrapped on cycles of the manifold. In section 3.6 we discuss wrapped D3 branes and study the four dimensional non-supersymmetric black holes they give rise to. We show that these non-supersymmetric black holes in type-IIB theory can be mapped to a string inside a deformed *brane-box* configuration again in type-IIB theory. We show that the deformation of the brane box appears from a B_{NS} field that seems to originate from the non-Kähler nature of the underlying six dimensional manifold. We also show that it might be possible to study the full supergravity background for the black hole using the brane-box/geometry correspondence.

In section 3.7 we study cosmological effects from brane-antibranes annihilations. We discuss many interesting effects here: separation of brane-antibranes due to the inflationary expansion and the resulting production of black holes, brane-antibranes annihilating to smaller branes and the resulting production of defects including black holes, etc.

In section 4, we classify all the possible solutions that one could, in principle, get by using our ansatz. We show that with two warp factors one could get de-Sitter cosmologies in type-IIB theory at most if the warp factors are all time-dependent. With three warp factors, we show that it might, a priori, be possible to get a de-Sitter space with a completely time-independent internal warp factor. We discuss possible consequences from moduli stabilization and interpret some of the recent advances in cosmological studies using our scenario.

In section 5 we use the results of section 3 and 4 to study primordial black holes in the D3/D7 system. In section 4 we showed that a de-Sitter kind of background might possibly exist using three warp factors in M-theory. In section 3.5, we gave the explicit metric for a particular internal space. We combine these observations and the results coming from wrapped branes and brane-antibrane annihilations to study possible primordial black holes. We argue that if the background can create copious numbers of brane-antibrane pairs, then one could possibly relate this to primordial black hole production in this set-up. In general, the number of such black holes, however, turns out to be almost negligible, and therefore it does not pose any cosmological problem. We use our earlier techniques to compare black hole formation in two different scenario: the original $K3 \times \mathbb{P}^1$ background and the new background that we derived in section 3.5. The former case gives zero productivity rates and the latter gives very small productivity rates (i.e., they are exponentially suppressed).

In section 6 we go to a different regime of our D3/D7 model. This regime need not necessarily correspond to an inflationary one.[8] Here we combine the background that we derived in section 3.5 with the F-theory picture to study semilocal defects on the D3 brane(s). We show that our model is rich enough to give us semilocal defects with almost all allowed global symmetries, including exceptional ones. Earlier studies of such defects were done at a group theoretical level in [56], where it was conjectured that defects with exceptional global symmetries should also exist. Here we show that our model might realize them as non-topological solitons of the theory. We will be able to provide explicit constructions of these defects using various techniques of algebraic geometry. Another interesting output of this analysis is a physical way to obtain the certain *homogeneous* quaternionic Kähler manifolds (in fact the ones that are symmetric spaces) directly from F-theory. This classification was done in the mathematical literature long ago [59, 60], and completed in [61]. Here we see their relevance to the existence of semilocal defects in the D3/D7 system.

This concludes the list of topics that we study in this paper. A more detailed discussion of cosmological solutions that are accelerating (possibly including de-Sitter type spacetimes) is left for the sequel to this paper [49].

2. Cosmological solutions from the $D3/D7$ system

In this section, we study cosmological solutions for the D3/D7 system from the full higher-dimensional viewpoint of ten- or eleven-dimensional string- or M-theory. In the earlier

[8] In other words, we will consider the regime where we can have more than two scalars. Although having more than two scalars do not necessarily rule out an inflationary description of the system, we will not concentrate too much on this aspect here.

papers [25, 30], this higher-dimensional viewpoint was only partially addressed, and the focus was instead more on the derivation of a four-dimensional effective action that describes the dynamics from a four-dimensional point of view. More precisely, it was shown that the presence of non-primitive fluxes on the world volume of D7-branes that are wrapped on a K3 manifold in a $K3 \times \mathbb{P}^1$ space would cause a D3-brane to move towards these D7-branes. The corresponding 4D effective theory was identified with a hybrid D-term (or, more generally, P-term) inflationary model, where the inflaton field, S, corresponds to the D3/D7 interbrane distance and the scalar field, $|\chi|$, whose condensation ends inflation, can be identified with a D3-D7 string mode that becomes tachyonic at a critical value of the D3/D7 interbrane distance. In order to study the full uplift of this scenario to ten or eleven dimensions, one now has to consider the following important points:

- A possible time dependence of the internal space.
- The noncompact 3+1 dimensional space as approximately a de-Sitter space.
- Moduli stabilization, at least, for the static case. This also includes radius stabilization so as to avoid the Dine-Seiberg runaway problem [62].

To account for the first point, we have to allow an ansatz that could have time dependent warp factors. However, having a time dependent warp factor would in general mean that the dynamics that depends on the volume of the internal spaces will now acquire a time dependence as well. This aspect can be used to put some strong constraints on the warp factors of the internal manifold.

The second point is the aspect which we would like to concentrate on the most. When the D3-brane starts moving towards the D7-branes, the initial background would approximately be a de Sitter space. In the 4D field theory, this corresponds to the slow-roll stage, where $|\chi|$ is trapped in its false minimum, and the inflaton field, S, is slowly rolling down its potential. At a critical point, $S = S_c$, when the D3 is very close to the D7-branes, $|\chi|$ becomes tachyonic and starts the waterfall stage where it quickly rolls down from its false vacuum to the final supersymmetric minimum, thereby ending inflation.

This picture is simpler and maybe a bit more transparent when viewed from M-theory. For the backgrounds we consider, this would be an M-theory compactification on a fourfold with G-fluxes. The initial condition is fixed by choosing G-fluxes that are not primitive. The final state of primitive G-fluxes (which in the language of inflation would be the final state after the waterfall stage) is obtained from the contributions of G-fluxes that are localized at the singularities of the fourfold (see [30] for details).

The third issue, the moduli stabilization, is by far the most difficult one. It is known that switching on fluxes can stabilize all complex structure moduli [63 – 65] at tree level. The Kähler moduli, including the radial modulus, on the other hand, are not stabilized by such fluxes. The first example, where the radial modulus was shown to be stabilized, was the heterotic compactification on a non-Kähler manifold [66]. For the type IIB case, it was realized that non-perturbative effects can stabilize the radial modulus [35]. For the other Kähler moduli, the proposal seems to be that a mixture of perturbative and non-perturbative effects can fix all the Kähler moduli [67] (see also [68] for the problems related to models with one Kähler modulus).[9]

But this is not all. In type-IIB (or in the heterotic string) there are the vector bundle moduli. In type-IIB they appear from the D7-branes, and they satisfy the Donaldson-Uhlenbeck-Yau equations. In M-theory, they would appear from the moduli of the G-fluxes that are localized at the orbifold singularities. It was shown in [71] that near these singularities, the G-fluxes decompose as a product of two two-form fluxes, one of them being a normalizable (1,1) form. Combining this with the primitivity condition of the G-fluxes, one reproduces precisely the DUY equations [71].

In any case, it seems to be highly non-trivial in general to construct complete models with all string theory moduli fixed at phenomenologically useful values (see e.g. [72] for some early attempts in the heterotic case). Therefore, the first step would be to fix all the moduli at least for the static case. We will briefly discuss the issue of moduli fixation later in section 4. First let us try to understand the basic cosmology of these models.

2.1 Basic cosmology of the model

Let us start with some standard ingredients from cosmology (see e.g. [73] [74] [75]). Homogeneous and isotropic cosmological scenarios in $d+1$ dimensional spacetimes are based on Friedmann-Lemaitre-Robertson-Walker (FLRW) type metrics. A spatially flat[10] $(d+1)$-dimensional FLRW line element has the form

$$ds^2 = -d\tau^2 + a^2(\tau) dx^i dx^i \,. \tag{2.1}$$

The sign of the Hubble parameter, $H \equiv \frac{\dot{a}}{a}$, determines whether the universe is expanding ($H > 0$) or contracting ($H < 0$). An expanding universe is accelerating if $\frac{\ddot{a}}{a}$ is positive and decelerating if $\frac{\ddot{a}}{a}$ is negative. During inflation, we have, by definition,

$$H > 0 \,, \qquad \frac{\ddot{a}}{a} > 0 \,. \tag{2.2}$$

Note that, for the proper evaluation of H and $\frac{\ddot{a}}{a}$, (2.1) should be the $(d+1)$-dimensional *Einstein frame* metric. (2.1) is conformally flat and can be written in terms of conformal time, $t \equiv x_0$, as

$$ds^2 = \eta^2(t) \left(-dt^2 + dx^i dx^i\right) = \eta^2(t)\, \eta_{\mu\nu} dx^\mu dx^\nu \,, \tag{2.3}$$

where t and $\eta(t)$ are defined by

$$\eta^2(t) dt^2 = d\tau^2 \,, \qquad \eta^2(t(\tau)) = a(\tau)^2 \,. \tag{2.4}$$

Eq. (2.4) implies $\eta dt = \pm d\tau$, and using this, one derives

$$\begin{aligned} H \equiv \frac{1}{a} \cdot \frac{da}{d\tau} &= \pm \frac{1}{\eta^2} \cdot \frac{d\eta}{dt} \\ \frac{1}{a} \cdot \frac{d^2 a}{d\tau^2} &= \frac{1}{\eta^2} \cdot \frac{d^2 \log|\eta|}{dt^2} \,. \end{aligned} \tag{2.5}$$

[9] Since we submitted our paper, important progress has been made in the construction of models with all moduli fixed. We refer to [69] for a very recent summary of this new work and many original references. Some earlier work in which some aspects of models with all moduli fixed were discussed include [70].

[10] For simplicitly, we restrict ourselves to spatially flat cosmologies.

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In other words, H can always be chosen positive by choosing the appropriate sign (which just reflects the invariance of the Einstein equations under time reversal), but the sign of the acceleration parameter $\frac{\ddot{a}}{a}$ is independent of this choice and given by the sign of $\frac{d^2 \log|\eta|}{dt^2}$. An important special case, which we will also encounter in some of our models below, is a simple power law behavior of the form

$$\eta(t) = ct^{\gamma} \tag{2.6}$$

with some, possibly fractional, power γ. It is easy to see from (2.5) that an expanding universe is accelerating precisely if

$$\gamma < 0 \qquad \Longrightarrow \qquad \frac{\ddot{a}}{a} > 0 \tag{2.7}$$

with a de Sitter space corresponding to the special case $\gamma = -1$.

In this section we are interested in uplifting accelerating spacetimes of the above type to compactifications of string or M-theory, having in mind a precise higher-dimensional realization of the D3/D7-brane inflationary scenario of type-IIB string theory containing higher derivative terms in the effective action.

Note that from a four-dimensional point of view, one could ask questions on the possible kinds of effective matter field configurations that give rise to such cosmological solutions. For instance, a perfect fluid with equation of state $p = w\rho$ gives rise to a time-scaling $a(\tau) \sim \tau^{[2/3(1+w)]}$ for the scale factor in (2.1) (see e.g. [73, 74]), with the energy density scaling as $\rho \sim a(\tau)^{-3(1+w)}$. In particular, this means $w = -1$ is a negative pressure fluid corresponding to a cosmological constant. Note also $w = +1$ corresponding to $p = \rho$ has been argued for as the "stiffest" equation of state [76]. In this light, it is conceivable that some of the cosmological solutions obtained from the higher dimensional lift naively exhibit exotic behaviour from the point of view of standard four-dimensional cosmology. Care must be used on the interpretation and validity of an effective four dimensional description of the cosmology as opposed to the full higher dimensional description (as we will discuss in section 3.3).

The construction of cosmological scenarios from higher dimensional theories has been addressed in many earlier papers [77, 78]. The basic idea followed in these papers [77] is to consider a compactification of a $(1 + d + D)$-dimensional theory and study the time dependent warped metric. The metric ansätze that have mostly been considered in the literature are of the form[11]

$$g_{MN} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & e^{2\tilde{A}}\delta_{ij} & 0 \\ 0 & 0 & e^{2\tilde{B}} g_{mn}(y) \end{pmatrix}, \tag{2.8}$$

where $x^i, x^j = 1, \ldots, d$ and $y^m, y^n = d+1, \ldots, D$, and the warp factors $\tilde{A}$ and $\tilde{B}$ have usually been assumed to be functions of cosmological time τ only. In this paper, we choose

[11] More general cases with different time-dependent warp factors for different factors of the internal space are of course also possible and have also been considered in various papers. We will return to this more general case with several internal warp factors in section 2.2. In the present subsection, we limit ourselves to the simplest possible case with only one internal warp factor, in order to illustrate the basic features of these models with the minimal amount of complexity.

to work with the conformal time $t \equiv x^0$ and, in general, also allow for a y-dependence of the warp factors, i.e., we will use ansätze of the form

$$ds^2 = e^{2A(y,t)}\left(-dx_0^2 + \sum_{i=1}^{d} dx_i dx^i\right) + e^{2B(y,t)} g_{mn}(y)\, dy^m dy^n \tag{2.9}$$

where $g_{mn}(y)$ is still not been specified, and $A(y,t)$ and $B(y,t)$ are the new warp factors that we shall use throughout (unless mentioned otherwise). In this form there is, an overall conformal factor in front of the $(d+1)$-dimensional part of spacetime. The total spacetime dimension, $(1+d+D)$, will either equal 10 or 11. For the special case of only time-dependent warp factors,

$$A(y,t) \equiv A(t)\,, \qquad B(y,t) \equiv B(t)\,, \tag{2.10}$$

the non-vanishing components of the Ricci tensor for the metric ansatz (2.9) are given by

$$R_{00} = -d\ddot{A} - D(\ddot{B} + \dot{B}^2 - \dot{A}\dot{B}) \tag{2.11}$$

where the dots are used to represent derivatives with respect to x_0, and

$$\begin{aligned} R_{ij} &= \left[\ddot{A} + D\dot{A}\dot{B} + (d-1)\dot{A}^2\right]\delta_{ij} \\ R_{mn} &= R^{(g)}_{mn} + e^{2(B-A)}\left[\ddot{B} + (d-1)\dot{A}\dot{B} + D\dot{B}^2\right] g_{mn}\,, \end{aligned} \tag{2.12}$$

where $R^{(g)}_{mn}$ is the Ricci tensor for the unwarped internal metric. Using $R = R_{MN}g^{MN}$, it is straightforward to work out the Einstein tensor $G_{MN} \equiv R_{MN} - \frac{1}{2}g_{MN}\, R$ from the above equations and to equate it to the energy momentum tensor T_{MN}. Using the ansatz (2.9), the curvature scalar is given by

$$R = e^{-2B}R^{(g)} + 2e^{-2A}\left[D\ddot{B} + d\ddot{A} + \frac{1}{2}D(1+D)\dot{B}^2 + D(d-1)\dot{A}\dot{B} + \frac{1}{2}d(d-1)\dot{A}^2\right] \tag{2.13}$$

where $R^{(g)}$ is the curvature scalar for the unwarped internal metric. For a Ricci flat internal manifold, the curvature scalar will only have contributions from the warp factors A and B. For the analysis above we took the warp factors to be time-dependent only. In general, they will be functions of y^m and t, and the Ricci tensors will get additional contributions, which we will display later. For the purpose of this paper, we will assume that the time- and y^m-dependences of the warp factors A and B factorize in the following way:

$$A(y,t) = \frac{1}{2}\left[\log f(y) + \log p(t)\right], \qquad B(y,t) = \frac{1}{2}\left[\log k(y) + \log h(t)\right]. \tag{2.14}$$

The factorization of $A(y,t)$ obviously separates out a $(d+1)$-dimensional FLRW metric written in conformal time as in (2.3), and is therefore desirable. It is also easy to see that in the case of a supersymmetric compactification, the time independence, $p(t) = h(t) = 1$, brings the warp factors in the form given in [79]. Another argument in favor of (2.14) can be given for the case where $h(t) = 1$. In this case, as we will discuss in the next section, there is an anomaly condition that is consistent *iff* the metric ansatz is of the form (2.14). For non-trivial $h(t)$, the choice of the warp factors (2.14) can isolate the time dependences from all the background equations of motion, as will become clear soon. Hence, the assumed factorization (2.14) is highly desirable.

Obviously, a non-trivial $h(t)$ would mean that the internal manifold has some time dependence. As mentioned earlier, there would then in general be a subtle issue in defining the concept of moduli stabilization (especially the volume modulus). We will come back to this point later. Instead, let us first recall another important consequence of such a non-trivial time-dependence. Given (2.14) in (2.9), the metric simplifies to

$$ds^2 = f(y)p(t)\, \eta_{\mu\nu}dx^\mu dx^\nu + k(y)h(t)\, g_{mn}(y)dy^m dy^n\,. \tag{2.15}$$

If (2.15) is in the $(1+d+D)$-dimensional Einstein frame, then the $(1+d+D)$-dimensional Einstein-Hilbert action can be integrated over the D compact dimensions to reduce to an action in $(1+d)$ spacetime dimensions of the form

$$K \int d^{d+1}x\, h^{\frac{D}{2}} \sqrt{-\det(g_{\mu\nu})}\; R^{(d+1)} + \cdots \tag{2.16}$$

where $R^{(d+1)}$ is the curvature scalar of the metric $g_{\mu\nu} \equiv p(t)\eta_{\mu\nu}$ in $(d+1)$ dimensions, and K is an integral over the internal D dimensions. The dotted terms denote the action for various fields that come from dimensional reduction of the Einstein term. In the notations used in (2.15), K is given explictly by:

$$K = \int d^D y\, f^{\frac{d+1}{2}}\, k^{\frac{D}{2}} \sqrt{\det(g_{mn})}\,. \tag{2.17}$$

K can always be absorbed by an overall rescaling of the coordinates x^μ and is irrelevant for the signs of H and $\frac{\ddot{a}}{a}$. The occurence of $h^{\frac{D}{2}}$ in (2.16), however, has to be properly taken care of by switching to the $(d+1)$-dimensional Einstein frame metric,

$$g^E_{\mu\nu} = h^{\frac{D}{d-1}}\, g_{\mu\nu} = h^{\frac{D}{d-1}}\, p(t)\, \eta_{\mu\nu} \qquad \Longrightarrow \qquad \eta^2(t) = h^{\frac{D}{d-1}}\, p(t)\,, \tag{2.18}$$

where $\eta^2(t)$ is now the proper scale factor of the proper $(1+d)$-dimensional Einstein frame metric (2.3). Furthermore, when the higher dimensional action has dilaton factors with the Einstein term, the metric in lower dimension will also pick up dilaton dependences. Combining (2.18) with the dilaton factors will reproduce the right Einstein frame metric for our case. However, for most of the analysis in this paper, the dilaton will in fact be zero, and therefore we can ignore the dilaton dependences altogether. In case the functions $p(t)$ and $h(t)$ are simple powers,

$$p(t) = at^\alpha\,, \qquad h(t) = bt^\beta\,, \tag{2.19}$$

we have $\eta^2(t) = c^2 t^{2\gamma} = c^2 t^{\alpha + \frac{\beta D}{d-1}}$, and hence,

$$\frac{\beta D}{d-1} + \alpha < 0 \qquad \Longrightarrow \qquad \frac{\ddot{a}}{a} > 0 \tag{2.20}$$

will be our condition for an accelerating universe. We shall use this in section 3 and 4 when we study some toy cosmologies in lower dimensions.

In the following, we will naturally be interested in the $3+1$ dimensional cosmology which corresponds to a compactification of the type-IIB theory to four spacetime dimensions. As we will see, however, the uplift to M-theory turns out to provide some simplifications and requires some different values for d and D. Furthermore, we will allow for a fairly general time dependence of the internal manifold. This is partially also motivated by the uplift to M-theory, where the internal manifold generically may have a time dependence even if the IIB internal manifold is completely static.[12]

So far, we have only concentrated on the metric, which we have treated in a way general enough to encompass all string theories as well as eleven-dimensional M-theory. However, to study promising cosmological models, one needs to switch on fluxes too. In M-theory, the only available fluxes are fluxes from the three form potential C. In type-IIB, by contrast, one can have a variety of fluxes: the RR and NS three forms, H_{NS} and H_{RR}, as well as the five form, F_5, so unlike the metric, the flux part should be analyzed case by case. In this paper, our basic model corresponds to the D3/D7 system of type IIB string theory on a six dimensional manifold with H_{NS} and H_{RR} fluxes switched on.

There are, however, different approaches how this model can be studied. For example, one can study the cosmology of this model directly in type-IIB theory. Alternatively, one can lift the whole configuration to M-theory and study the cosmology from there. The latter approach doesn't mean that one takes $d=3$ and $D=7$. In fact, the lift to M-theory described in [25] corresponds to $d=2$ and $D=8$. This would mirror the type-IIB dynamics with $d=3$ and $D=6$. The M-theory approach has some clear advantages over the type-IIB case. In M-theory all the D7-branes and O7-planes become pure orbifold singularities of a torus fibration. The type-IIB fluxes, on the other hand become G-fluxes in M-theory. In M-theory, one would therefore have to concentrate only on the dynamics of a complex fourfold with G-fluxes.[13] M-theory on such a fourfold with G-fluxes has been well studied starting with [79, 64]. In [79], it was noted that, if we allow an ansatz (2.9) for the metric with $d=2$ and $D=8$, then the allowed form of the G-flux would be

$$G = \partial_m \, e^{3D} \, dy^m \wedge dx^\mu \wedge dx^\nu \wedge dx^\rho + \frac{1}{4!} \partial_{[\alpha} \, C_{npq]} \, dx^\alpha \wedge dy^n \wedge dy^p \wedge dy^q \tag{2.21}$$

where $D(t, y^m)$ is another warp factor,[14] $x^\alpha \equiv (x^0, y^m)$, and C_{npq} is the three form completely in the internal space. One might wonder whether one could have a simple config-

[12]Other scenarios in which the internal manifold is time-dependent by construction include [6] or the models of [80] or the racetrack inflation model of [17] where the inflaton is actually the volume modulus. In a slightly different vein, time dependent seven dimensional internal manifold that is used to study four-dimensional cosmologies has recently been addressed in [48].

[13]One might worry whether type eleven dimensional supergravity remains a good description when the fiber torus becomes very small. As we discuss soon, we will consider various higher derivative corrections that depend on the curvatures and fluxes in M-theory. Since the fiber can be made conformally flat over a large region of a six dimensional base, the quantum corrections coming from torus size can be controlled.

[14]In this form, the ansatz is slightly different from the one chosen by [79] in the static and supersymmetric case. More precisely, we made the simplest extension to the non-supersymmetric case, namely, the fields C_{012} and C_{mnp} are now allowed to be functions of (y^m, t) instead of just y^m, as in the supersymmetric case. A more generic ansatze in which we switch on other components like $C_{\mu mn}$ or $C_{\mu\nu m}$ probably could also be entertained but we will not do so here.

uration where this flux is switched off. It turns out, however, that if one does not allow any M2 branes, then we cannot have vanishing three form for a compact fourfold [79, 81] as there would otherwise be an uncancelled anomaly from the non-trivial Euler number of the fourfold. For the non-compact case, this is no longer true. This also means that in type-IIB we necessarily need the internal three form(s) (as we plan to keep only a small countable number of D3 branes). Another important point is that supersymmetry would require the internal G-fluxes to be primitive [79]. On the other hand, if we do not require supersymmetry, then the G-fluxes (2.21) are not constrained by primitivity. This will be clear soon when we discuss the background equations of motion.

With this in mind, we now consider the low energy limit in eleven dimensions with leading quantum corrections and an $M2$ brane (i.e the membrane) source term. This is the direct lift of the type-IIB background to M-theory. As is well known, the precise map of the type-IIB data to M-theory are[15]

- The threefold of type-IIB becomes a fourfold in M-theory. The fourfold has non-zero Euler number because the torus fibration is non-trivial.
- The NS and RR fluxes of type-IIB will simply become G-fluxes in M-theory. These G-fluxes are spread over the full fourfold.
- The seven-branes and orientifold seven-planes all become singularities of the T^2 fibration of the fourfold, i.e., the T^2-fiber degenerates over the six dimensional base. Note, however, that the total fourfold itself is a *smooth* eight-dimensional manifold.
- The seven-brane gauge fluxes become *localised* G-fluxes in M-theory. These localised fluxes appear at the singularities of the T^2-fibration only and have zero expectation values away from the singularities [71].
- The D3-brane will become an M2 brane in M-theory. The cosmology of the model will therefore consist of M2 brane(s) moving towards orbifold singularities of the torus fibration of the fourfold. Supersymmetry is broken by choosing non-primitive G-fluxes.

To study this in detail, we need the full action in M-theory. The action will consist of three pieces: a bulk term, $S_{\rm bulk}$, a quantum correction term, $S_{\rm quantum}$, and finally a membrane source term, S_{M2}. The action is then given as the sum of these three pieces:

$$S = S_{\rm bulk} + S_{\rm quantum} + S_{M2}\,. \tag{2.22}$$

The individual pieces are:

$$S_{\rm bulk} = \frac{1}{2\kappa^2}\int d^{11}x\sqrt{-g}\left[R - \frac{1}{48}G^2\right] - \frac{1}{12\kappa^2}\int C\wedge G\wedge G\,, \tag{2.23}$$

[15]Some aspect of this was discussed earlier in [82]. One of the author (KD) would also like to thank C. Herdeiro and S. Hirano with whom related investigation of these backgrounds were performed without taking quantum corrections into account.

where we have defined $G = dC$, with C being the usual three form of M-theory, and $\kappa^2 \equiv 8\pi G_N^{(11)}$. This is the bosonic part of the classical eleven-dimensional supergravity action. It is clear that this is not enough, as we require quantum corrections to the system, otherwise we cannot put fluxes or branes in this scenario (see [79, 81] for more details). The leading quantum correction to the action can be written as:

$$S_{\text{quantum}} = b_1 T_2 \int d^{11}x\sqrt{-g}\left[J_0 - \frac{1}{2}E_8\right] - T_2\int C\wedge X_8\,, \tag{2.24}$$

where the expressions for J_0, E_8 and X_8 were already given in [79, 25].[16] The coefficient T_2 is the membrane tension. For our case, $T_2 = \left(\frac{2\pi^2}{\kappa^2}\right)^{\frac{1}{3}}$, and b_1 is a constant number given explicitly as $b_1 = (2\pi)^{-4}3^{-2}2^{-13}$. The M2 brane action is now given by:

$$S_{M2} = -\frac{T_2}{2}\int d^3\sigma\sqrt{-\gamma}\left[\gamma^{\mu\nu}\partial_\mu X^M\partial_\nu X^N g_{MN} - 1 + \frac{1}{3}\epsilon^{\mu\nu\rho}\partial_\mu X^M\partial_\nu X^N\partial_\rho X^P C_{MNP}\right], \tag{2.25}$$

where X^M are the embedding coordinates of the membrane (recall that the membrane is along the 2+1 dimensional spacetime, and therefore is a point on the fourfold). The world-volume metric $\gamma_{\mu\nu}, \mu,\nu = 0,1,2$ is simply the pull-back of g_{MN}, the spacetime metric. Due to the last term, the motion of this M2 brane is obviously influenced by the background G-fluxes. The geodesic motion of the M2 brane

$$\Box X^P + \gamma^{\mu\nu}\partial_\mu X^M\partial_\nu X^N\Gamma^P_{MN} = r_o\,, \tag{2.26}$$

where $\Box$ is the laplacian operator and

$$r_o \equiv \frac{1}{3!}\;\epsilon^{\mu\nu\rho}\partial_\mu X^M\partial_\nu X^N\partial_\rho X^Q G^P{}_{MNQ}\,, \tag{2.27}$$

is the standard one when $r_o = 0$. In that case Γ^P_{MN} would be the standard Christoffel symbols. However because of the background G-fluxes, this is not the case and r_o becomes non-zero and will then govern the motion of the M2 brane in this scenario.

The background field equations consist of two sets of equations:

$$\begin{aligned} &R^{MN} - \frac{1}{2}g^{MN}R = T^{MN} && : \text{Einstein Equation} \\ &d * G = \frac{1}{2}G\wedge G + 2\kappa^2(T_2X_8 + *J) && : \text{G-flux Equation}\,. \end{aligned} \tag{2.28}$$

[16]Although not required for this paper, the definition of J_0, X_8 and E_8 are:

$$\begin{aligned} J_0 &= 3\cdot 2^8\;\; R^{MNPQ}R_{KNPS}R_M{}^{TUK}R^S{}_{TUQ} + \frac{1}{2}R^{MNPQ}R_{KSPQ}R_M{}^{TUR}R^S{}_{TUN} \\ E_8 &= \frac{1}{6}\epsilon^{ABCN_1\ldots N_8}\epsilon_{ABCM_1\ldots M_8}R^{M_1M_2}{}_{N_1N_2}R^{M_3M_4}{}_{N_3N_4}R^{M_5M_6}{}_{N_5N_6}R^{M_7M_8}{}_{N_7N_8} \\ X_8 &= \frac{1}{3\cdot 2^9\cdot\pi^4}\;\;\mathrm{tr}\,R^4 - \frac{1}{4}(\mathrm{tr}\,R^2)^2\;\;. \end{aligned}$$

Thus, E_8 is the eleven dimensional generalization of the Euler integrand. Furthermore, the epsilon symbol appearing here is a tensor and not a tensor density. Therefore, with all its indices upper, this would be proportional to $\frac{1}{\sqrt{-g}}$ while with all its indices lower, it will be proportional to $\sqrt{-g}$. This will be crucial later when we have to extract the warp factor dependences of various components.

The energy-momentum tensor appearing in the Einstein equation has three pieces. The first is the standard contribution from G-fluxes, given by:

$$T_G^{MN} = \frac{1}{12}\left[G^{MPQR}G^N{}_{PQR} - \frac{1}{8}g^{MN}G^{PQRS}G_{PQRS}\right]. \tag{2.29}$$

The second contribution comes from the quantum corrections. This is also easy to work out, and is given by

$$T_Q^{MN} = \frac{b_1\kappa^2 T_2}{\sqrt{-g}}\frac{\partial}{\partial g_{MN}}\left[\sqrt{-g}(2J_0 - E_8)\right], \tag{2.30}$$

where all the quantities have been defined earlier. The third, and final, contribution to the energy-momentum tensor comes from the membrane term. Since the membrane is localised at a point on the fourfold, we would require delta functions to describe its position. The membrane term is given explicitly by

$$T_B^{MN} = -\frac{\kappa^2 T_2}{\sqrt{-g}}\int d^3\sigma\sqrt{-\gamma}\gamma^{\mu\nu}\partial_\mu X^M\partial_\nu X^N\delta^{11}(x-X), \tag{2.31}$$

where we used $\delta^{11}(x-X)$ to denote the position of the M2 brane on the fourfold. Since we expect the membrane to be moving, these delta functions will themselves be time dependent via $\delta^{11}(x - X(t))$. We will discuss the implications of this when we study the solutions explictly.

The second equation in (2.28) is the equation for G-fluxes. This can be written in terms of components as

$$\begin{aligned} D_M(G^{MPQR}) = \epsilon^{PQRM_1....M_8}&\left[\frac{1}{2\cdot(4!)^2}G_{M_1...M_4}G_{M_5...M_8} + \frac{2\kappa^2 T_2}{8!}(X_8)_{M_1....M_8}\right] + \\ &+\frac{2\kappa^2 T_2}{\sqrt{-g}}\int d^3\sigma\sqrt{-\gamma}\ \epsilon^{\mu\nu\rho}\partial_\mu X^P\partial_\nu X^Q\partial_\rho X^R\delta^{11}(x-X) \end{aligned} \tag{2.32}$$

where the last term is the definition of J[17] that we used in (2.28), and D_M is the covariant derivative. Furthermore, in addition to (2.32), there would be the Bianchi identity for the G-fluxes.[18]

It is worth mentioning that spacetime metrics of the form (2.3), (2.9) are necessarily only applicable in an effective supergravity description, which can potentially break down in some regions. For instance, singularities in the warp factors $A(t), B(t)$ can potentially develop at specific locations in the time t-coordinate, where one must necessarily resort to a more microscopic description. Obvious locations for such singularities are the endpoints of the time t-interval where such a spacetime metric is treated as valid: e.g. we expect a breakdown towards the end of the slow-roll phase when the D3-brane makes contact with the 7-branes, signalling the dominance of possible stretched open string tachyonic

[17] J should not be confused with the quantity J_0.

[18] The equations that we presented here will get further corrected by higher derivative terms, which are in *addition* to the quantum terms that we mentioned here. The necessity of these terms will become clear when we will try to construct explicit solutions to these equations.

modes (in the type-II description). But also noteworthy is the potential breakdown of the above solutions when (in the M-theory lift) the M2-brane is far from any singularities of the fourfold, since the higher derivative terms in the M-theory lift are likely to be less important here and the M2-brane only senses the local approximately trivial local geometry of the fourfold.[19] Thus the gravity solutions we will find in what follows are necessarily to be understood within these regimes of validity and this will in fact be borne out in the specifics below.

2.2 Analysis of the background equations of motion

Having set up the equations, it is now time to consider a little more realistic model. We have already mentioned several general features of the lift of the $D3/D7$ system to M-theory. For the particular choice of $K3 \times T^2/\mathbb{Z}_2$ (or, more generally, $K3 \times \mathbb{P}^1$ if one is away from the orientifold limit) as the six-dimensional compact space in type-IIB theory with D3- and D7-branes, this lift to M-theory was made more explicit in [25]. There it was argued that the M-theory compactification will be on an eight-dimensional manifold which is $K3 \times K3$. The fluxes in the $D3/D7$ system that are responsible to break supersymmetry in the Coulomb phase become G-fluxes in M-theory. The D7-brane that is wrapped on the base K3 and is a point on the $\mathbb{P}^1$ simply disappears in M-theory and is absorbed in the other K3. Recall that a K3 manifold is a non-trivial T^2 fibration over a $\mathbb{P}^1$ base. This $\mathbb{P}^1$ is the same $\mathbb{P}^1$ on which our D7-brane in the IIB picture is a point. The axion-dilaton, which is seeded by the D7-brane(s), becomes the complex structure of the T^2-fiber and degenerates precisely at the loci of the seven-branes on the $\mathbb{P}^1$. This way, we can account for the complicated brane system of the type-IIB framework by using a simple (smooth) compact manifold. The D3-brane, as we already mentioned, becomes an M2 brane.

This is of course only one out of many possible backgrounds. In section 3.5 we will also discuss another consistent background with D3/D7-branes that allows for certain black hole configurations from wrapped branes. The M-theory lift of this is another fourfold and will be discussed in section 3.5. The manifold $K3 \times K3$ has $b_3 = 0$ whereas the other background has a non-vanishing third Betti number $b_3 \geq 1$. Both of these backgrounds have explicit F-theory descriptions and therefore they are consistent static (and hence supersymmetric) supergravity solutions, that may even be quantum-mechanically exact.

The time dependent case, on the other hand, is a slightly more tricky issue. The reason is that the warp factors will not only depend on time t, but also on the internal coordinates y^m. The metric ansätze that we want to consider for this case will be slightly more generic than (2.9) in the sense that we will allow for three warp factors $A = A(t, y^m), B = B(t, y^m)$

[19]For example, as we will encounter later in section 3, for early times (small t) we may have some components of the metric vanishing and G_{0mnp} diverging with C_{012} vanishing. While for late times (large t), we may have divergent metric warping with G_{omnp} vanishing and C_{012} diverging (for instance, this example will appear in section (3.1)). Both ends at the level of the supergravity solution seem unreliable or imcomplete but the M-theory physics is not singular (just the two ends of the M2-brane motion). This of course doesn't mean that supergravity description is breaking down, rather that we have to be somewhat careful in interpreting the physics here.

and $C(t, y^m)$, i.e

$$ds^2 = e^{2A}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + e^{2B}\,g_{mn}\,dy^m dy^n + e^{2C}\,|dz|^2\,, \tag{2.33}$$

$\mu, \nu = 0\ldots2$, $m, n = 4\ldots9$, and $dz = dx^3 + \tau dx^{11}$. Here, dz is the coordinate of the fiber torus with complex structure τ, and the coordinates of the fourfold are: $y^{4,5\ldots9}, x^3, x^{11}$, where x^3 comes from T-dualizing the third coordinate in the IIB picture and x^{11} is the M-theory circle. The Ricci tensors will now depend on both the time and the space indices. The unwarped metric g_{mn} is a function of y^m only and will be taken to be independent of time t. Similarly, τ will depend on y^m only. In other words, the only time-dependence is assumed to come from the warp factors A, B and C. The case where some components of the unwarped metric themselves are time-dependent, is equivalent to having a different ansatz for the metric (than the one that we made above in (2.33)) where also more time-dependent warp factors for the internal space could exist (for example three or more warp factors for the internal space). Of course such an even more general ansatz cannot be ruled out and a detailed discussion will be presented in the sequel to this paper.

The above ansatz with three warp factors can be understood as a special case of the two warp factor case (2.9) if the internal, unwarped metric in (2.9) is allowed to depend on time as well. More precisely, defining $y^{a,b} = x^3, x^{11}$, we can rewrite the warp factors as

$$e^{2C}\;|dz|^2 = e^{2B}\,g_{ab}\,dy^a dy^b\,, \tag{2.34}$$

which is nothing more than saying that g_{ab} is time independent. Using the above relation, physics with two warp factors A, B in (2.9) and three warp factors A, B, C in (2.33) will be exactly the same. An alternative way to say this would be to observe that as long as

$$C = B + \log\sqrt{g_{33}}, \qquad \text{and} \qquad \partial_0 C = \partial_0 B \tag{2.35}$$

there is no difference between (2.9) and (2.33). The difference comes when the time evolution of C and B are not the same. In that case the theory will have three different warp factors. This will happen, for example, when g_{33} itself is time-dependent (which is equivalent to saying that we have a different warped torus fiber). More generic discussion will be presented below.

When the internal manifold is a flat torus i.e when $g_{mn} = \delta_{mn}$ with a square fiber two-torus, and the warp factors only depend on the internal space coordinates, the Ricci tensors (with two warp factors) have been worked out in [83]. For the generic case of an additional time dependence (and with three warp factors) the Ricci tensor along the non-compact spacetime directions is given by:

$$\mathcal{R}_{\mu\nu} = -\eta_{\mu\nu}e^{2A-2B}\left[\Box A + 3\partial_m A\partial^m A + 4\partial_m A\partial^m B + 2\partial_m A\partial^m C\right] + R_{\mu\nu}\,, \tag{2.36}$$

where $R_{\mu\nu} = R_{00}, R_{ij}$, $(i, j = 1, 2)$ are more complicated than the ones that we derived earlier in (2.11) and (2.12), respectively, due to the presence of the third warp factor. They are now given by

$$\begin{aligned} R_{ij} &= \left[\ddot{A} + 6\dot{A}\dot{B} + \dot{A}^2 + 2\dot{A}\dot{C}\right]\delta_{ij} \\ R_{00} &= -2\ddot{A} - 6(\ddot{B} + \dot{B}^2 - \dot{A}\dot{B}) - 2(\ddot{C} + \dot{C}^2 - \dot{A}\dot{C})\,. \end{aligned} \tag{2.37}$$

The Ricci tensor along the m, n directions will likewise pick up both time and space derivatives and is given by

$$
\begin{aligned}
\mathcal{R}_{mn} = & \, 3\left[2\partial_{(m}A\partial_{n)}B - \partial_m A\partial_n A - g_{mn}\partial_k A\partial^k B\right] + 4\left[\partial_m B\partial_n B - g_{mn}\partial_k B\partial^k B\right] - \\
& -3D_{(m}\partial_{n)}A - 2D_{(m}\partial_{n)}C + 2\left[2\partial_{(m}C\partial_{n)}B - \partial_m C\partial_n C - g_{mn}\partial_k C\partial^k B\right] - \\
& -4D_{(m}\partial_{n)}B - g_{mn}\Box B + R^{(g)}_{mn} + e^{2(B-A)}\left[\ddot{B} + \dot{A}\dot{B} + 6\dot{B}^2 + 2\dot{C}\dot{B}\right] g_{mn} \qquad (2.38)
\end{aligned}
$$

where $R^{(g)}_{mn}$ is the "unwarped" Ricci tensor of the six dimensional base of the fourfold and D_m is the covariant derivative with respect to the unwarped internal metric g_{mn}. An easy check of the above relations is to plug in

$$
g_{mn} = \delta_{mn}\,, \qquad \tau = i\,, \qquad \dot{A} = \dot{B} = \dot{C} = 0\,, \qquad B = C \qquad (2.39)
$$

assuming that the warp factors depend only on the internal coordinates, which reproduces the Ricci tensors of [83] with two warp factors. In addition to the above set of components, there are now also components of the Ricci tensor that do not appear when the warp factors are independent of time or when we have two warp factors as in (2.9). These are the Ricci tensors $\mathcal{R}_{0m}$ and $\mathcal{R}_{ab}$. They are given by

$$
\begin{aligned}
\mathcal{R}_{0m} &= 7\dot{B}\partial_m A + 2\dot{B}\partial_m C + 2\dot{C}\partial_m A - 2\dot{C}\partial_m C - 2\partial_m\dot{A} - 5\partial_m\dot{B} - 2\partial_m\dot{C} \\
\mathcal{R}_{ab} &= -\delta_{ab}e^{2(C-B)}\left[\Box C + 3\partial_m C\partial^m A + 4\partial_m C\partial^m B + 2\partial_m C\partial^m C\right] + \\
& \quad + e^{2(C-A)}\left[\ddot{C} + \dot{A}\dot{C} + 6\dot{C}\dot{B} + 2\dot{C}^2\right] \qquad (2.40)
\end{aligned}
$$

where we have taken $\tau = i$ to evaluate $\mathcal{R}_{ab}$. Its easy to generalize to arbitrary complex structures. One notes that the $\mathcal{R}_{0m}$ components will not be visible for the case when we take the warp factors to be independent of y^m but dependent on time t. Similarly, if there are only two warp factors, the terms $\dot{B}\partial_m C - \dot{C}\partial_m C$ in $\mathcal{R}_{0m}$ would cancel when $B = C$. Therefore such terms are only visible for more than two warp factors. With four warp factors there would likewise be even more terms. The Ricci scalar is now easy to evaluate from the tensors given above. It is given explicitly by

$$
\begin{aligned}
\mathcal{R} = & -e^{-2B}\left[10\Box B + 6\Box A + 2\Box C + 20\partial_m B\partial^m B\right] - 3e^{-2B}\left[4\partial_m A\partial^m A + 6\partial_m A\partial^m B\right] - \\
& -2e^{-2B}\left[3\partial_m C\partial^m C + 8\partial_m B\partial^m C + 6\partial_m A\partial^m C\right] + e^{-2B}\, R^{(g)} + \\
& +2e^{-2A}\left[6\ddot{B} + 2\ddot{A} + 2\ddot{C} + 21\dot{B}^2 + 6\dot{A}\dot{B} + 12\dot{C}\dot{B} + 2\dot{A}\dot{C} + \dot{A}^2 + 3\dot{C}^2\right] \qquad (2.41)
\end{aligned}
$$

where $R^{(g)}$ is the Ricci scalar of the six dimensional base with metric g_{mn}. Since the fiber torus is a square one, it does not degenerate in local neighborhoods and therefore has vanishing curvature. This also means that the seven-branes are kept far away. Again, this is not the most generic picture, but it is simple enough to illustrate the basic structure.

Let us now come to the G-fluxes. We have decomposed the spacetime coordinates as $x^M = [x^\mu, y^m, y^a]$. The G-fluxes would consequently have to be divided along those directions because of the different warp factors. The G-fluxes are non-trivial functions of the fourfold coordinates as well as time, just as the warp factors. As a simple special case, we will, in section 3.1, consider in greater detail warp factors that depend only on time t. But before we come to this special case, let us first take a general look at the most generic case. For that it is convenient to write everything in terms of unwarped metric and unwarped fields. From the ansatz (2.33), we can easily see that the warped and the unwarped fields are related as follows:

$$
\begin{aligned}
&G^{012m} \to G^{012m}\, e^{-6A-2B}\,, \quad G^{012a} \to G^{012a}\, e^{-6A-2C}\,, \quad \det g \to e^{6A+12B+4C} \det g\\
&G^{0mnp} \to G^{0mnp}\, e^{-2A-6B}\,, \quad G^{0mna} \to G^{0mna}\, e^{-2A-4B-2C}\,, \quad G^{0mab} \to G^{0mab}\, e^{-2A-2B-4C}\\
&G^{mnpq} \to G^{mnpq}\, e^{-8B}\,, \quad G^{mnpa} \to G^{mnpa}\, e^{-6B-2C}\,, \quad G^{mnab} \to G^{mnab}\, e^{-4B-4C}
\end{aligned} \tag{2.42}
$$

where the unwarped fields are on the right. We will also concentrate on the case where the world volume coordinates $\sigma^{0,1,2}$ are identified with $x^{0,1,2}$. The embedding coordinates y^m and y^a, however, are no longer constants, as we expect the M2 brane to move on the fourfold. Thus,

$$
\gamma_{\mu\nu} = e^{2A}\eta_{\mu\nu} + e^{2B}\partial_\mu y^m\, \partial_\nu y^n\, g_{mn} + e^{2C}\partial_\mu z \partial_\nu \bar{z}\,, \tag{2.43}
$$

where we have isolated the unwarped metric components.

We remind the reader that the M-theory solution is only to mimic the real type-IIB solution that we are really interested in. From the choice of the metric (2.33) in M-theory, the type-IIB metric can be easily worked out. For our case, let us define a vector $\mathrm{x} = \begin{pmatrix} x^3 \\ x^{11} \end{pmatrix}$, which is the fiber coordinate, and denote the corresponding metric of the fiber torus as g^t. With these definitions, one has

$$
e^{2C}\,|dz|^2 \equiv e^{2\tilde{C}}\, d\mathrm{x}^\top g^t\, d\mathrm{x} \qquad \Longrightarrow \qquad C = \tilde{C} + \log\sqrt{\mathrm{tr}(g^t\sigma_1)}\,, \quad \tau = \frac{\mathrm{tr}(g^t\sigma_4) + i\sqrt{\det g^t}}{\mathrm{tr}(g^t\sigma_1)}\,, \tag{2.44}
$$

where σ_i are the Chan-Paton matrices (see for example eq. (3.24) of [84]).[20] Observe also that (2.44) is more precise than (2.34) and (2.35), as it encodes the information of non-trivial complex structure (i.e the fiber torus is not a square one) in a concise way. Furthermore, writing the metric in terms of warp factor $\tilde{C}$ instead of C has the advantage that $\tilde{C} = B$ will be the case of two warp factors. We will eventually work out the two warp factor case in detail later. For the present purpose, it will be instructive to write the type-IIB metric we get from M-theory with three warp factors (2.33). This type-IIB metric turns out to be:

$$
ds^2 = e^{2A+C}|\tau|\left(-dx_0^2 + dx_1^2 + dx_2^2\right) + \frac{e^{-3C}\,|\tau|}{(\mathrm{Im}\,\tau)^2}\, dx_3^2 + e^{2B+C}|\tau|\, g_{mn}\, dy^m dy^n\,, \tag{2.45}
$$

which tells us that the metric of the internal space in M-theory could in principle be time dependent even when the internal metric in IIB string theory is not. Both the axion $\tilde{\phi}$ and the dilaton field ϕ for this background will in general not vanish as we have not removed all singularities far away. In fact the axion-dilaton $(\tilde{\phi}, \phi)$ is generated by the non-trivial complex structure of the fiber torus as

$$
\varphi = \tilde{\phi} + ie^{-\phi} = \frac{\tau}{|\tau|^2}\,, \tag{2.46}
$$

[20] In this language $\det g^t = \mathrm{tr}(g^t\sigma_1)\cdot \mathrm{tr}(g^t\sigma_3) - \mathrm{tr}(g^t\sigma_2)\cdot \mathrm{tr}(g^t\sigma_4)$.

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which tells that that for a square torus, $\tau = i$, there would be no axionic field.[21] In addition to this, the G-fluxes can allow NS fluxes in type-IIB theory, and this would break supersymmetry. Thus (2.45) will be the expected metric that we would get here, and along with the M-theory G-fluxes (2.21) − which will give us the non-primitive threeforms − will specify the full background.

From the type-IIB metric (2.45) we observe something interesting. The metric along the $x^{0,1,2}$ directions, in general, behaves differently from the metric along the x^3 direction. The difference comes from the three different warp factors that we have allowed for the M-theory metric. We can therefore have the following four scenarios:

$$\begin{aligned}
&(1)\ A, B = \text{arbitrary}\,, && C = B + \log\sqrt{\mathrm{tr}(g^t\sigma_1)}\\
&(2)\ A + 2C + 2\log\mathrm{Im}\,\tau = 0\,, && C = B + \log\sqrt{\mathrm{tr}(g^t\sigma_1)}\\
&(3)\ A, B, C = \text{arbitrary}\\
&(4)\ A + 2C + 2\log\mathrm{Im}\,\tau = 0\,, && B = \text{arbitrary}\,.
\end{aligned} \tag{2.47}$$

The first two are basically two warp factor cases, and the next two are genuine three warp factor cases. The first and the third case arise when we study the cosmology of the model without taking all quantum corrections into account. The second case, will tell us that we have a 3+1 dimensional space that has an overall warp factor of $\frac{e^{-3C}\,|\tau|}{(\mathrm{Im}\,\tau)^2}$. As will be clear soon, this case is an exact solution for the supersymmetric background. All the background equations show miraculous cancellations when we apply this condition. Whether this condition persists for the time dependent case is still not clear. This case might arise when we take the full quantum corrections into account. We will provide some evidence for this in this paper. A more detailed analysis will be left for the sequel to this paper. The fourth case, with a special relation between A and C, is much less constrained than case 2. It has an advantage over case 2 from the fact that this might keep the internal six manifold in type-IIB to be time-independent. On the other hand, an increase in number of warp factors makes the analysis of background equations of motion much more difficult. Furthermore, it is also not clear whether the simple ansatz for the G-flux with two warp factors (2.21) can carry over to the three warp factors case. In this paper, therefore, we will be mostly concerned with two warp factors, although we will perform some analysis with three warp factors also. For the two warp factor cases, it would simplify quite a bit if we also move the seven-branes far away and only study the motion of the D3-brane(s) in a non-primitive background. This means that the axion vanishes, i.e., we consider the fiber torus to have a metric $g^t = \mathrm{diag}[g_{33}\ g_{11,11}]$. For this approximation, the type-IIB metric will become:

$$ds^2 = e^{2A+B}\sqrt{g_{11,11}}\left(-dx_0^2 + dx_1^2 + dx_2^2\right) + \frac{e^{-3B}}{g_{33}\sqrt{g_{11,11}}}dx_3^2 + e^{3B}\sqrt{g_{11,11}}\,g_{mn}\,dy^m dy^n \tag{2.48}$$

[21]It is easy to see that when the torus is non-trivially fibered over the base (with coordinates y^m), we expect to get a cross-term g_{3m} in the metric (2.45). Existence of such cross terms have been proposed recently in a paper [85] (although with only two warp factors) that appeared after our work. Here, in this paper, we will stick with the simplest possible ansatze for finding cosmological solutions. In the sequel to this paper, more generic ansatze will be taken.

with an almost vanishing axion-dilaton.[22] Note that for constant metric components of the T^2 fiber of the fourfold, the second condition of (2.47) (with $g^t = \mathbb{I}$) will become $A \approx -2B$. This simplification will have some important consequences as we will soon see.

Let us first study the background G-fluxes. The equations of motion for the G-fluxes are given by (2.32). To study them, we will allow the following non-trivial components of G-fluxes

$$G_{012m}\,, \qquad G_{0mnp}\,, \qquad G_{mnpq} \tag{2.49}$$

where, by a slight abuse of notation, m, n denote *all* the coordinates of the internal space.[23] Using the scaling relations given in (2.42) and the first condition of (2.47), the equation governing the component G_{mnpq} is given by

$$\partial_0\left(e^{A+2B}G^{0mnq}\right) = D_p\left[e^{3A}(G^{mnpq} - (*G)^{mnpq})\right] - \frac{2\kappa^2 T_2}{8!}\epsilon^{mnpa_1\ldots a_8}(X_8)_{a_1\ldots a_8} \tag{2.50}$$

where $a_1 \ldots a_8 = 0, 1, 2\ldots$ and therefore involve coordinates outside the fourfold[24] that are essentially conformally flat. Therefore they have vanishing curvature tensors. Now since the X_8 term is measured w.r.t. to the curvatures, this can be made arbitrarily small and we can ignore it completely. On the other hand, when X_8 has all the components inside the fourfold, this will no longer be the case because of non-trivial curvatures, and then X_8 will contribute.

The covariant derivatives and the Hodge star in (2.50) are all measured with respect to the unwarped metric. We now observe a few interesting details from the above equation. The first is that the coefficient of G^{0mnq} is e^{A+2B}. Using our simplifying second condition of (2.47) for a square torus, this would simply go away. And therefore (2.50) will be an equation for $\partial_0 G^{0mnq}$. The second observation is that, for the case when the warp factors and the G-fluxes are independent of time t, the fluxes satisfy the self-duality condition

$$G_{mnpq} = (*G)_{mnpq} \tag{2.51}$$

which is exactly the primitivity condition. As is known from [79], having primitivity is equivalent to having supersymmetry. Therefore (2.50) tells us how we could break supersymmetry: a small deviation from primitivity (2.51) will break supersymmetry for our case. We therefore define

$$G_{mnpq} - (*G)_{mnpq} \equiv e^{-3A}\gamma_{mnpq} \neq 0 \tag{2.52}$$

[22]The dilaton, or $e^{-\phi}$, is given by $\mathrm{Im}\,\tau$ (2.46). If we choose the fiber components such that $g_{11,11} \approx g_{33}$, then the dilaton can also be made very small.

[23]These are the coordinates of the fourfold and hence $m, n = 4\ldots 9, 3, 11$ unless mentioned otherwise. When we go to type-IIB, $m, n = 4\ldots 9$. It should always be clear from the context whether we mean a fourfold or a six dimensional base. Similarly the metric g_{mn}, Ricci-scalar $R^{(g)}$ etc. are either fourfold or six-manifold data depending on whether we are in M-theory or type-IIB.

[24]We have kept X_8 as a function of the *warped* metric, as we are not very concerned about its scaling behavior with respect to the warp factors A, B in this paper. Everything else is written in terms of the unwarped metric. Furthermore, the identity $\int X_8 = -\frac{1}{4!(2\pi)^4}\chi$ with χ being the Euler number of the fourfold, continues to hold because the warp factors are all globally defined variables and therefore do not affect the integral.

as our condition for breaking supersymmetry. The quantity γ would therefore determine the scale of susy breaking here. The above consideration actually imposes the following two conditions on the G-flux components G_{0mnq}:

$$D_m G^{0mnq} = 0\,, \qquad \partial_0 G^{0mnq} = D_p \gamma^{mnpq}\,. \tag{2.53}$$

The first condition implies covariant constancy of G^{0mnq}, and the second implies susy breaking by G-fluxes. Thus, choosing a covariantly constant function on the fourfold will give the spatial part of G^{0mnq}. The temporal part of G^{0mnq}, i.e the susy breaking parameter γ, on the other hand can be related to the other components of the G-fluxes, the warp factors as well as the membrane velocities. The precise relation can be worked out with some effort and is given by:

$$e^{-6B} G^{mnpq} G_{0npq} + \frac{12\kappa^2 T_2 e^{-6B}}{\sqrt{-g}} \int d^3\sigma \dot{y}^m\, \delta^{11}(x-X) + 6e^{-6B}\partial_0\partial^m e^{6B} - G^m = 0 \tag{2.54}$$

where we have defined $G^m \equiv \gamma^{mnpq} G_{0npq}$. We have also taken $D = -2B$ in (2.21). The equation (2.54) can be used to relate the membrane velocities to the warp factors, when γ and G_{mnpq} are known. Therefore, to determine the membrane velocities, we need to know the warp factors completely as they are again intertwined with G-fluxes. The equation for the warp factor can be determined from (2.32) by taking the equation of motion for the component G_{012m}, as:

$$-\Box e^{6B} = \frac{1}{2\cdot 4!} G_{mnpq} (*G)^{mnpq} + \frac{2\kappa^2 T_2}{\sqrt{-g}} \left[\delta^8(y-Y) + \frac{X_8}{8!}\right] \tag{2.55}$$

where $\Box$ in the l.h.s. comes from applying condition (2) of (2.47) to (2.32) and (2.42). Integrating the above equation reproduces the anomaly relation for the fourfold, implying that we can only put fluxes on a compact fourfold if we also consider quantum corrections. For the non-compact fourfold, this poses no problem of course.

So far, we have found that the component G_{0mnq} can be determined by knowing the susy breaking function γ. The other component $G_{012m} = \partial_m e^{3D}$ will be known from the solution of (2.55) and the metric g_{mn} once we specify the relation between D and B. From the relation (2.52) it is clear that once we determine the Hodge operator $*$ we can use (2.52) to fix the value of G_{mnpq}. The equations that relate the metric components g_{mn} with the fluxes and warp factors are the Einstein equations (2.28). These are very complicated second order equations, and therefore let us tackle them component by component. We start with the 00 component: $G_{00} = T_{00}$. The generic form of G_{00} is given by:

$$\begin{aligned} G_{00} = e^{2(A-B)}\Big[& d(2-D)\partial_m A\partial^m B - \frac{d(1+d)}{2}\partial_m A\partial^m A - \frac{(D-2)(D-1)}{2}\partial_m B\partial^m B - \\ & - d\,\Box A + \frac{R^{(g)}}{2} - (D-1)\Box B\Big] + \frac{d(d-1)}{2}\dot{A}^2 + \frac{D(D-1)}{2}\dot{B}^2 + Dd\dot{A}\dot{B} \end{aligned} \tag{2.56}$$

where $d = 2$ and $D = 8$. The scalar curvature of the fourfold is given by $R^{(g)}$. The Einstein equation will become

$$
\begin{aligned}
e^{2(A-B)}\Bigg[&\frac{R^{(g)}}{2} - 12\partial_m A\partial^m B - \frac{21}{4}\partial_m A\partial^m A - 21\partial_m B\partial^m B - \\
&- 2\,\Box A - 7\,\Box B\Bigg] + \dot{A}^2 + 28\dot{B}^2 + 16\dot{A}\dot{B} = \\
&= \frac{e^{-6B}}{4!}G_{0mnp}G^{0mnp} + e^{2A-8B}\Bigg(\frac{1}{4\cdot 4!}G_{mnpq}G^{mnpq} - \\
&\qquad - \frac{\kappa^2 T_2 e^{-A}}{\sqrt{-g}}\int d^3\sigma\sqrt{-\gamma}\,\gamma^{00}\,\delta^{11}(x-X)\Bigg) + \\
&+ \frac{\kappa^2 b_1 T_2 e^{-3A-8B}}{2\sqrt{-g}}\frac{\partial}{\partial A}\Big[\sqrt{-g}(2J_o - E_8)e^{\mathrm{f}}\Big]
\end{aligned}
\tag{2.57}
$$

where b_1 is a constant defined earlier, $\mathrm{f} \equiv a_1 A + a_2 B$ where $a_{1,2}$ are constants that can be determined from the metric components and γ^{00} is a non-trivial function of the membrane velocities. The term $\frac{21}{4}\partial_m A\partial^m A$ in (2.57) comes from the contribution of the energy momentum tensor $G_{012m}G^{012m}$, and the last term in (2.57) comes from quantum corrections. If we define $E_q \equiv \sqrt{-g}(2J_0 - E_8)e^{\mathrm{f}}$, then the quantum corrections to the energy momentum tensor fall into the following three categories:

$$
\mathcal{T}_1 = \frac{\partial E_q}{\partial A}, \qquad \mathcal{T}_2 = \frac{\partial E_q}{\partial B}, \qquad [\mathcal{T}_3]_{mn} = \frac{\partial E_q}{\partial g_{mn}}\,. \tag{2.58}
$$

Therefore even for $g_{mn} = \delta_{mn}$, there would be contributions from the quantum corrections coming from the warp factors A and B. For the specific choice of our metric, all the $\mathcal{T}_i$ are non-zero.

Let us now analyze the equation (2.57). We can plug in the values of G_{0mnp} and G_{mnpq} which we determined earlier. The equation (2.57) therefore intertwines $A, B, \dot{y}^m$ and g_{mn}, where $\dot{y}^m$ enters through the definition of γ and γ^{00} given earlier in (2.43). The scalar curvature is of course zero for a Ricci flat fourfold. Observe also that we haven't yet imposed the relation $A = -2B$ on the above equation. Before we do that, we can re-arrange this equation a little bit by using the warp factor equation (2.55). The warp factor equation is written in terms of $\Box e^{6B}$ on the l.h.s. . In terms of A and B, the l.h.s. of (2.55) can be replaced by

$$
\Box e^{6B} \;\leftrightarrow\; -3\,D_m\left(e^{6B}\partial^m A\right) \tag{2.59}
$$

with the r.h.s. of (2.55) unchanged. Plugging (2.59) into (2.55), we can add the equations (2.57) and (2.55) in the following way:

$$
2\Big[\text{eq. (2.57)}\Big] - e^{2A-8B}\Big[\text{eqn (2.55)}\Big]\,, \tag{2.60}
$$

so that the final equation for our background will become:

$$
e^{2(A-B)}\Bigg[R^{(g)} - 42\left(\partial_m A\partial^m B + \frac{1}{4}\partial_m A\partial^m A + \partial_m B\partial^m B\right) -
$$

$$
\begin{aligned}
&\left. - 7(\Box A + 2\,\Box B)\right] + 2\dot{A}^2 + \\
&+56\dot{B}^2 + 32\dot{A}\dot{B} = \\
&\quad = \frac{e^{-6B}}{2\cdot 3!} G_{0mnp}G^{0mnp} + \frac{e^{2A-8B}}{2\cdot 4!}\left[G_{mnpq}G^{mnpq} - G_{mnpq}(*G)^{mnpq}\right] + \\
&\quad + \frac{2e^{2A-8B}\kappa^2 T_2}{\sqrt{-g}}\left[\left(\frac{b_1 e^{-5A}}{2}\frac{\partial E_q}{\partial A} - \frac{X_8}{8!}\right) - \right. \\
&\qquad\qquad \left. - \int d^3\sigma\left(e^{-A}\sqrt{-\gamma}\gamma^{00} + 1\right)\delta^{11}(x - X)\right]. \qquad (2.61)
\end{aligned}
$$

The above equation may look even more complicated than the one that we started with, i.e (2.57). However if we look closely, we see that once we impose the condition $A = -2B$ the equation (2.61) shows some amazing simplification. Putting $A = -2B$ in (2.61) results in the following equation:

$$
\frac{1}{12}G_{0mnq}G^{0mnq} + \frac{1}{48}\,G_{mnpq}\gamma^{mnpq} + e^{-6B}T_2\mathcal{Q} = 0 \qquad (2.62)
$$

where $\mathcal{Q}$ is the last term of (2.61), which is a combination of the quantum and the membrane term. In deriving this we have strictly used $A = -2B$. This is true only when $g_{33}g_{11,11} =$ constant. This however is not true generically. Therefore we will get additional warp factor dependent terms in (2.62) when we use $A = -2B - \log\sqrt{g_{33}g_{11,11}}$ as the relation between the warp factors.

In any case, the above equation (2.62) can only tell us the y^m dependent terms of the warp factor. For the complete description we need the time dependences of the warp factor. This can in principle come from the other Einstein equations. The G_{ij}, $i, j = 1, 2$ components can be written as:

$$
\begin{aligned}
G_{ij} \;=\; \delta_{ij}\, e^{2(A-B)}&\left[d(D-2)\partial_m A\partial^m B + \frac{d(d+1)}{2}\partial_m A\partial^m A + d\,\Box A + (D-1)\,\Box B + \right. \\
&\left. - \frac{R^{(g)}}{2} + \frac{(D-2)(D-1)}{2}\partial_m B\partial^m B\right] + \qquad (2.63) \\
+\delta_{ij}&\left[(1-d)\ddot{A} - D\ddot{B} - \frac{D(D+1)}{2}\dot{B}^2 - \frac{(D-1)(D-2)}{2}\dot{A}^2 - D(d-2)\dot{A}\dot{B}\right].
\end{aligned}
$$

Putting in the values of $d = 2$ and $D = 8$, we can get the Einstein equation $G_{ij} = T_{ij}$, for $i, j = 1, 2$. Since x^1, x^2 are on the same footing, the equation for any of these components is:

$$
\begin{aligned}
e^{2(A-B)}&\left[12\partial_m A\partial^m B + \frac{21}{4}\partial_m A\partial^m A + 2\,\Box A + 7\,\Box B + \right. \\
&\left. + 21\partial_m B\partial^m B - \frac{R^{(g)}}{2}\right] - \ddot{A} - 8\ddot{B} - 36\dot{B}^2 = \\
&\quad = \frac{e^{-6B}}{4!}\,G_{0mnq}G^{0mnq} - \frac{b_1\kappa^2 T_2 e^{-3A-8B}}{2\sqrt{-g}}\,\frac{\partial E_q}{\partial A} - \qquad (2.64) \\
&\quad - \frac{e^{2A-8B}}{4\cdot 4!}G_{mnpq}G^{mnpq} - \frac{\kappa^2 T_2 e^{A-8B}}{\sqrt{-g}}\int d^3\sigma\sqrt{-\gamma}\,\gamma^{ii}\,\delta^{11}(x - X).
\end{aligned}
$$

Comparing (2.64) with (2.57) we see some crucial differences. The space derivatives of the warp factors are exactly negative of each other, and so is the quantum correction. On the other hand, the G^2_{0mnq} term comes with the same sign, and the membrane term has γ^{ii} (no sum over i) instead of γ^{00} as in (2.57). This tells us that we can add these two equations to get the following simple relation:

$$
\begin{aligned}
[\text{eq. (2.57)}] + [\text{eq. (2.64)}] &= \dot{A}^2 - 8\dot{B}^2 + 16\dot{A}\dot{B} - \ddot{A} - 8\ddot{B} \\
&= \frac{e^{-6B}}{12} G_{0mnq}G^{0mnq} - \frac{\kappa^2 T_2 e^{A-8B}}{\sqrt{-g}} \times \\
&\quad \times \int d^3\sigma \sqrt{-\gamma}(\gamma^{00} + \gamma^{ii})\, \delta^{11}(x - X)
\end{aligned} \tag{2.65}
$$

which relates the time derivatives of the warp factors to the G-fluxes G_{0mnq} and the membrane velocities. We now make three observations related to (2.65). The first is that in the static gauge (where we identify the membrane coordinates with $x^{0,1,2}$), $\gamma^{00} + \gamma^{ii} = 0$ (again no sum over i indices), and therefore the membrane contribution would vanish. For our case this term is non vanishing precisely because of the membrane velocities. Secondly, (2.65) will be one equation where the quantum effects are cancelled out. We will soon exploit this property to get some cosmological solutions. And finally, since the l.h.s. of (2.65) involves only time derivatives the $\sqrt{g_{33}g_{11,11}}$ term in $A = -2B - \log\sqrt{g_{33}g_{11,11}}$ will drop out, and therefore the l.h.s. will become $-6(\ddot{B} + 6\dot{B}^2)$. This will be useful soon.

Most of the above analysis has focussed on the time and space derivatives of the warp factors A and B with the explicit metric of the internal fourfold appearing only through higher order corrections or/and through possible membrane terms. A more direct way to get the metric would be to concentrate on the G_{mn} term. This is given explicitly as

$$
\begin{aligned}
G_{mn} = {} & G^{(g)}_{mn} + (d+1)\left[2\partial_{(m}A\partial_{n)}B - \partial_m A\partial_n A - D_{(m}\partial_{n)}A\right] + \\
& +(D-2)\left[\partial_m B\partial_n B + D_{(m}\partial_{n)}B\right] + \\
& +g_{mn}\Bigg[(d+1)\Big(\Box A + (D-3)\partial_k A\partial^k B + \\
& \qquad + \frac{d+2}{2}\partial_k A\partial^k A\Big) + (D-2)\left(\Box B + \frac{D-3}{2}\partial_k B\partial^k B\right)\Bigg] - \\
& -e^{2(A-B)}g_{mn}\Bigg[(D-1)\ddot{B} + d\ddot{A} + \frac{D(D-1)}{2}\dot{B}^2 + (D-1)(d-1)\dot{A}\dot{B} + \\
& \qquad + \frac{d(d-1)}{2}\dot{A}^2\Bigg]
\end{aligned} \tag{2.66}
$$

where $G^{(g)}_{mn}$ is the Einstein tensor for the unwarped metric g_{mn}. For the fourfold in consideration, the equation of motion resulting from the above will be:

$$
\begin{aligned}
& G^{(g)}_{mn} - 3\left[D_{(m}\partial_{n)}A + 2D_{(m}\partial_{n)}B\right] + \\
& +6\left[\partial_{(m}A\partial_{n)}B + \frac{1}{4}\partial_m A\partial_n A + \partial_m B\partial_n B\right] +
\end{aligned}
$$

$$
\begin{aligned}
&+g_{mn}\left[3\,\Box A+6\,\Box B+15\left(\partial_k A\partial^k B+\frac{1}{4}\partial_k A\partial^k A+\partial_k B\partial^k B\right)\right]-\\
&-e^{2(B-A)}g_{mn}\left(7\ddot{B}+2\ddot{A}+28\dot{B}^2+7\dot{A}\dot{B}+\dot{A}^2\right)=\\
&\qquad=\frac{e^{-6B}}{12}\left[G_{mpqr}G_n{}^{pqr}-\frac{g_{mn}}{8}G_{pqrs}G^{pqrs}\right]-\\
&\qquad-\frac{e^{-2A-4B}}{4}\left[G_{0mpq}G_n{}^{0pq}-\frac{g_{mn}}{6}G_{0pqr}G^{0pqr}\right]+\\
&\qquad+\frac{b_1\,\kappa^2\,T_2e^{-3A-10B}}{\sqrt{-g}}\left[\frac{1}{2}(g^{-1})_{mn}\mathcal{T}_2+[\mathcal{T}_3]_{mn}\right]-\\
&\qquad-\frac{T_2\kappa^2}{\sqrt{-g}}e^{-3A-4B}\int d^3\sigma\sqrt{-\gamma}\,\gamma^{00}\dot{y}^p\dot{y}^q\,g_{mp}\,g_{nq}\,\delta^{11}(x-X)
\end{aligned}
\tag{2.67}
$$

where the $\mathcal{T}_i$ are defined in (2.58). We see that all the $\mathcal{T}_i$ are responsible for the total energy momentum tensor in this scenario (because of the non-trivial warp factors and metric components). Thus along with the membrane contribution, (2.67) will determine the unwarped metric components once all others are known.

Looking at (2.67) we see that the equation can get drastically simplified once we impose the relation between the two warp factors $A \approx -2B$. For the Ricci flat fourfold, all the spatial derivatives of A and B go away completely in the limit $A = -2B$. Of course this simplifying relation is not true generically, and therefore we will eventually be left with some spatial derivatives. For the time being, as above, let us concentrate only on $A = -2B$. The l.h.s. of (2.67) will now become $-3g_{mn}\,e^{6B}(\ddot{B}+6\dot{B}^2)$. Observe that the time derivatives of B is exactly the same as we got earlier for (2.65). On the other hand (2.65) doesn't have any quantum terms whereas (2.67) does. Therefore by contracting (2.67) with metric g_{mn} we can have non-trivial relations between the quantum (and possibly the membrane) terms. It will be interesting to relate the resulting equation to (2.62). We will however not exploit these details in this paper and leave the rest of the discussion for the sequel. The cosmological solutions that we will study in the next two sections will ignore some of the quantum effects.

The above set of equations would have sufficed if the warp factors were only space or time dependent. This is obviously not the case here and therefore we will have yet another equation that would relate the space-time derivatives of the warp factors. Recall that so far we had either space derivatives or the time derivatives (or the sum of both) of the warp factors i.e $\partial_m\partial_n A$ or $\ddot{A}$ etc. Now we expect equations that relate $\partial_m\dot{A}$ terms. In fact there is a non-trivial Einstein tensor that does the job for us. This is G_{0m} and is given by

$$
G_{0m}=(d+D-1)\dot{B}\partial_m A-d\partial_m\dot{A}-(D-1)\partial_m\dot{B}\,, \tag{2.68}
$$

where m, as usual, are all the internal coordinates. The Einstein equation resulting from this can be easily derived from the requisite energy-momentum tensor. It is given explicitly as:

$$
9\dot{B}\partial_m A-2\partial_m\dot{A}-7\partial_m\dot{B}=\frac{e^{-6B}}{2\cdot 3!}\,G_{0pqr}G_m{}^{pqr}- \tag{2.69}
$$

$$-\frac{e^{-A-6B}\kappa^2 T_2}{\sqrt{-g}}\int d^3\sigma\sqrt{-\gamma}\,\gamma^{00}\dot{y}^n\,g_{mn}\,\delta^{11}(x-X)$$

which has vanishing quantum terms because of the absence of cross terms in the metric of the form g_{0m}. In the presence of such cross terms the above equation (2.69) will have an additional quantum term. When we consider $A = -2B$ the l.h.s. of (2.69) becomes $-3(\partial_m\partial_0 B + 6\partial_0 B\partial_m B)$ which is then related to the G-fluxes and the membrane velocities via (2.69).

The equation (2.69) can now be compared to (2.54). Both these equations relate the membrane velocities to the warp factors and G-fluxes. To see whether there is any connection between the two equations, we have to rewrite (2.54) in a more suggestive way, as:

$$3\partial_0(e^{6B}\partial^m A) = \frac{1}{3!}\,(*G)^{mnpq}G_{0npq} + \frac{2\kappa^2 T_2}{\sqrt{-g}}\int d^3\sigma\,\dot{y}^m\,\delta^{11}(x-X)\,. \tag{2.70}$$

In this form this equation is close to (2.69). In the limit $A = -2B$, the l.h.s. of (2.70) becomes $-6(\partial_m\partial_o B + 6\partial_o B\partial_m B)$. Thus,

$$\begin{aligned}[\text{eq. (2.70)}] - 2e^{6B}\,[\text{eq. (2.69)}] &= \frac{1}{3!}\,G_{0npq}\,[(*G)^{mnpq} - G^{mnpq}]\\ &= \frac{\kappa^2 T_2}{\sqrt{-g}}\,e^{2(B-A)}\,\dot{y}^m\,|\dot{y}|^2\,\delta^8(y-y(t))\,. \end{aligned}\tag{2.71}$$

Comparing this equation with (2.52), we see that if the l.h.s. is zero, then the membrane velocities have to vanish. This is perfectly consistent with what we know, namely: when (2.52) vanishes, we have *primitive* G-fluxes. Therefore supersymmetry will be preserved, and so the membrane cannot move. Breaking primitivity via switching on a non-zero γ_{mnpq} will immediately trigger the motion of the M2 branes from (2.71). Therefore we need a non-zero G_{mnpq} satisfying (2.52) with non-vanishing γ_{mnpq} to start the motion of the M2 brane in our setup. The non-zero G_{mnpq} on the other hand should be (a) quantized, and (b) time independent. The former condition is the requirement first proposed in [86], and later elaborated on in numerous papers including [64]; and the latter condition is from the warp factor equation (2.55). It is encouraging to see how the expected conditions appear from the background equations of motion.

With this set of equations, the background can, in principle, be completely determined. As we saw above, the background equations of motion are pretty involved not only because they are second order differential equations, but also because they involve quantum corrections that are fourth order in curvature. Solving these equations therefore becomes complicated. In the next few sections we will try to solve these equations by ignoring some of the quantum and the membrane terms. Again because of the anomaly constraint (2.55) it is not always possible to ignore quantum corrections. We have to consider things case by case and see how far we can ignore these corrections. The cases we will concentrate on here are the ones given in (2.47) which considers only two warp factors A and B. As mentioned earlier, this is by no means a generic choice. In fact we will see that with two warp factors we get cosmological examples, that are interesting, but not quite a de-Sitter one. Therefore, it could be that the dynamical evolution of the fields in this theory is given by three or more warp factors. We will provide some evidence here that with three warp factors we can get close to a de-Sitter cosmology, but the precise equations will not be worked out here. These and other detailed scenarios will be investigated in the sequel to this paper.

Another interesting question is to ask if there are general restrictions on the possible cosmologies (i.e. on the warp factors) that are allowed when the higher derivative terms are included, in particular stemming from the energy conditions that they impose (see section 3.3 for more on this). Furthermore it is tempting to draw potential parallels between this effective theory involving higher derivative terms in this cosmological setting and the microscopic entropy counting matches [87, 88] and string-corrections [89, 90] of 2-charge BPS black hole solutions in supergravity theories with higher derivative terms [91], including questions of possible field redefinitions therein [90].

3. Some toy cosmologies in detail

In the previous section, we have laid out the full set of equations that govern cosmological solutions of M-theory on a fourfold with time-dependent warp factors, fluxes and M2-branes. In this section we make a first attempt towards solutions of these equations by considering some simplified situations that reduce the complexities of the equations of motion. These toy cosmologies are not always meant to correspond to realistic cosmologies, but rather serve as means to discuss various important features of the more realistic scenarios in a simplified context and to gain some better intuition for these more realistic solutions.

Before we study these toy models, however, it is worth emphasizing an important difference to the cosmological solutions that have mostly been studied before. In order to see this, recall that our framework of M-theory on an fourfold (corresponding to $d = 2$ and $D = 8$) is chosen only as a tool to study the cosmology of the D3/D7 system in type-IIB theory on a threefold (corresponding to $d = 3$ and $D = 6$), which is what we are really interested in. This distinguishes our approach from direct studies of $(3 + 1)$-dimensional cosmologies (i.e., the case $d = 3$) in compactifications of six- or seven-dimensional spaces (i.e., $D = 6$ or $D = 7$) in string or M-theory, respectively, which appeared for example in [77]. Typical among those examples are Kasner type solutions or Jordan-Brans-Dicke type solutions [92] (see also [78] for further work on cosmological solutions in M- or string theory). For the type of solutions studied in [77], the typical behavior of the warp factor is

$$e^A = \sum_n a_n\, t^n\,, \qquad e^B = \sum_m b_m\, t^m\,, \qquad \text{with } n_{\max} > m_{\max}\,, \tag{3.1}$$

i.e., three spatial dimensions expand much faster than the remaining six/seven dimensions. In our case with $d = 2$ and $D = 8$, the situation is a bit more complicated, as one of the internal dimensions of the fourfold should correspond to the third expanding dimension in the type-IIB picture. The interpretation of the different warp factors therefore requires some care, as we will soon illustrate.

In the following examples we will study cosmologies when we switch on small internal fluxes, and also arbitrary internal fluxes. We will give a new six dimensional space that satisfies all the equations of motion and whose metric is completely determinable; along with cosmological effects from wrapped branes and brane-antibranes annihilations on this geometry. All these cases will mostly be with two warp factors. We shall give evidence that with three warp factors it might be possible to get a de-Sitter type-IIB cosmology.

3.1 Example 1: small background fluxes

In order to reduce the complexity due to the y^m- and t-dependence of the warp factors, we will start with a toy example where the warp factors only depend on time t. We will further assume that we have only two warp factors, i.e., we will consider case 1 of (2.47) with warp factors $A(t)$ and $B(t)$ as we are not putting any relation between the warp factors *a-priori*.

For this case, the M2 brane as usual will move on the fourfold. The equation of motion for the M2 brane is (2.26) with $\gamma_{\mu\nu}$ given by (2.43). In the case where the embedding coordinates of the membrane only depend on time t, the above relation (2.43) simplifies to

$$\gamma_{00} = -e^{2A} + e^{2B}\dot{y}^m\dot{y}^n\ g_{mn}\,, \qquad \gamma_{11} = \gamma_{22} = e^{2A}\,, \qquad \gamma_{12} = 0\,, \tag{3.2}$$

and therefore the world volume metric is no longer trivial as it would be in the static supersymmetric case. The motion of the M2 brane is triggered by non-primitive G-fluxes via (2.71). Therefore, we need a non-zero G_{mnpq} which is quantized and time independent that satisfies (2.52), and most importantly (2.55). For a G-flux to satisfy (2.55), we have to *necessarily* impose quantum corrections, because without quantum corrections, $G_{mnpq} = 0$. We also have to solve (2.50) for the component G_{0mnp}, and in general we would require G_{0mnp} to be time dependent. On the other hand, G_{mnpq} has to be time independent so as to satisfy (2.55), and also (2.52), to break supersymmetry. Furthermore, the integral of G_{mnpq} over *any* four-cycle has to be quantized as an integer or as a fraction depending on the type of background one chooses. This leads us to the following ansatz for the background three-form field in the internal space:

$$C = [\psi(t) + \lambda(y)]_{mnp}\ dy^m \wedge dy^n \wedge dy^p\,. \tag{3.3}$$

The G-fluxes consequently will be: $G_{0mnp} = \partial_0\psi_{mnp}$ and $G_{mnpq} = 4\partial_{[m}\lambda_{npq]}$. We also require

$$\frac{1}{2\pi}\int_{4-\text{cycle}} G_{mnpq}\, dy^m \wedge dy^n \wedge dy^p \wedge dy^q = \frac{\mathbb{Z}}{2}\,, \tag{3.4}$$

where $\mathbb{Z}$ is an integer. If we assume that all the four-cycles are large in the cosmological setting, then we can break supersymmetry by very small expectation values of the G fluxes. This means in particular that γ in (2.52) is $\gamma \sim c_3 e^{3A}$ with $c_3 \to 0$. The C field will now take the following form:

$$C = [\psi(t) + \epsilon\cdot y]_{mnp}\ dy^m \wedge dy^n \wedge dy^p, \qquad \epsilon \to 0 \tag{3.5}$$

and ϵ and c_3 can be related by (2.52). The case with γ not small will be dealt with later.

With this ansatz for the C field, the value of G_{0mnp} can be easily determined from (2.50). The r.h.s. of (2.50) has $D_p\gamma^{mnpq}$, and this vanishes because it is only a function of time (via the warp factor $A(t)$). In general, therefore, the internal components of the G-flux are determined solely by the warp factors appearing in the metric and a possible quantum correction as:

$$G^{0mnp} = e^{-A-2B} \int dt\, (D_p \gamma^{mnpq}) + e^{-A-2B} (c_o + c_1)^{npq} \tag{3.6}$$

where c_o is independent of time and c_1 is the quantum correction coming from the X_8 term in (2.50). Since some of the directions of X_8 are along $x^{1,2}$, $c_1 = 0$ for our case.[25]

Consider now the motion of the M2 brane on the fourfold. We expect the membrane to move slowly on the fourfold towards one of the orbifold singularities. Once we isolate a bunch of singularities at a point and keep the other singularities far away, the membrane will eventually fall into the singularities that are nearby. With the assumption that we made above, this is possible to realize. It is easy to see that if we ignore the quantum correction to the energy-momentum tensor T^{0m}

$$\frac{\partial}{\partial g_{0m}} (2J_0 - E_8) \approx 0 \tag{3.7}$$

and use (2.69) with $G_{mnpq} \to 0$ and (3.6), then the velocity of the membrane on the fourfold will be constrained as

$$\frac{\kappa^2 T_2}{\mathrm{V}_8} \frac{\dot{y}^m}{\sqrt{1-v^2}} = \frac{1}{2 \cdot 3!}\, G_{0npq}\, G^{mnpq}\,, \tag{3.8}$$

where V_8 is the volume of the fourfold, v is the warped velocity of the membrane given by $v = e^{B-A}\, |\dot{y}|$, and G^{mnpq} is proportional to ϵ as discussed above. Thus, the internal flux G_{mnpq} is a vanishingly small quantity. However, the r.h.s. of the above equation would not vanish when the warp factors are functions of the internal coordinates, or/and if we switch on large expectation values for G_{mnpq}. We will discuss these cases later, but for the time being let us assume (3.8) to hold. One possible way for (3.8) to hold without violating any of the conditions would be to assume a very slow velocity of the membrane on a fourfold of very large volume V_8, that is.[26]

$$\dot{y}^m \approx 0\,, \qquad \mathrm{V}_8 \to \infty\,. \tag{3.9}$$

This way we can take care of (3.8). The above equation would also mean that the contribution to the energy momentum tensor coming from the velocities of the membrane is almost negligible and therefore we can ignore it for the time being. Thus, (3.9) will be our slow roll condition. For the case when $A = -2B$, the slow roll condition can be equivalently put as

$$\frac{\kappa^2 T_2}{\mathrm{V}_8} \dot{y}^m\, v^2 = -\frac{1}{3!}\, G_{0npq}\, \gamma^{mnpq}\,. \tag{3.10}$$

[25] This quantum correction may not always vanish for the case when all components are inside the fourfold. But we can make this small by taking the fourfold as an orbifold, with orbifold singularities. The quantum corrections there will be determined from the number of singularities and will in fact be close to zero away from the singular points. If we take the base of the fourfold to be large and assume that the orbifold points are also shifted far away, then this can be made very small. This way we can keep the x^3 and x^{11} directions of the fiber to be small and still get a good type-IIB cosmology.

[26] The issue of large size fourfold is a little subtle, because it deals with moduli stabilization. We will discuss this later.

So far, we have the following situation: we have a large radius fourfold with a time dependent C_{mnp} flux switched on (the space dependent part of C_{mnp} is vanishingly small although the integral $\int G\wedge G$ over the full fourfold is a non-vanishing integer satisfying (2.55)). The membrane on the fourfold is moving very slowly towards the orbifold fixed point that we have isolated from the other bunches of singularities. The fluxes break supersymmetry and satisfy equations (2.50), (2.54), (2.71) and (2.55). We haven't discussed the issue of moduli stabilization as yet, and neither did we solve for the warp factors A and B, which we took to be explicit functions of time. From the analysis of the previous section, we know all the equations for the warp factors, and we will now study them one by one. The first non-trivial equation for the warp factors will be (2.57). From the ansatze of A, B and the solution (3.6), we see that (2.57) simplifies. However, (2.57) also has membrane and quantum terms that we cannot ignore. Furthermore, the membrane terms are written in terms of $\delta^8(y-y(t))$ where $y^m(t)$ denotes the position of the membrane on the fourfold at a given time t moving with a velocity $|\dot{y}|^2 = \dot{y}^m\dot{y}^n g_{mn}$. But the issue of membrane and the quantum terms can actually be ignored if we consider (2.61), where we used (2.60) to get from (2.57) to (2.61). In (2.61) we see that the quantum terms come as a *difference* between two terms. We can, therefore, assume that the difference can be made small even though the individual terms may not be very small.[27] Therefore the membrane and the quantum terms T_m and T_q respectively goes to zero as:

$$\begin{aligned} T_q &\equiv \frac{b_1 e^{-5A}}{2}\,\frac{\partial E_q}{\partial A} - \frac{X_8}{8!} \to 0 T_m^{(1)} \\ &\equiv \frac{1}{2}\, e^{2(B-A)}\ |\dot{y}|^2\, \delta^8(y-y(t)) \to 0\,, \end{aligned} \tag{3.11}$$

where we used (3.9) to derive the second equation. Under the above assumptions, we get the first non-trivial equation for the warp factors:

$$\dot{A}^2 + 28\dot{B}^2 + 16\dot{A}\dot{B} = \frac{e^{-6B}}{24}|G|^2 - \frac{1}{2}e^{2(A-B)}\ R^{(g)} + \frac{e^{2A-8B}}{4\cdot 4!}\ \mathcal{O}(\epsilon^2)\,, \tag{3.12}$$

where $R^{(g)}$ is the Ricci scalar of the fourfold, and we have defined $|G|^2 = G_{0mnp}G^{0mnp}$. Notice also that we have put in the contributions from the G_{mnpq} terms that are of the form $\frac{e^{2A-8B}}{4\cdot 4!}\ \mathcal{O}(\epsilon^2)$. A more precise computation, which we will do next, will have to incorporate this term, as well as the membrane and the quantum terms $T_m^{(1)}$ and T_q, respectively.

The next equation for the warp factor is (2.64). We see that this also has non-trivial membrane and quantum terms. However (2.65) gives us an equation where the quantum terms have cancelled out completely and the membrane term would go to zero as:

$$T_m^{(2)}\ \equiv\ e^{2B}\ |\dot{y}|^2\, \delta^8(y-y(t)) \to 0\,, \tag{3.13}$$

giving us the second non-trivial equation for the warp factors:

$$\dot{A}^2 - 8\dot{B}^2 + 16\dot{A}\dot{B} - \ddot{A} - 8\ddot{B} = \frac{e^{-6B}}{12}\ |G|^2\,. \tag{3.14}$$

[27] If the warp factors are not too big, then $\frac{\partial E_q}{\partial A}$ and $\frac{\partial E_q}{\partial B}$ could be made small in principle. Also the X_8 terms are sensitive to orbifold singularities. Therefore if we shift the singularities far away (this means the base of the fourfold is of very large radius) then we can also make X_8 small locally. In this case the difference T_q can indeed be very small.

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The above set of equations can be simplified a little bit by taking into account the fact that the unwarped metric of the fourfold is Ricci flat. As we saw earlier, this of course doesn't mean that the six-dimensional base also has to be Ricci flat. In fact, generically, the six dimensional base is never a Calabi-Yau manifold or even a Kähler manifold. Ricci flatness of the fourfold implies that

$$G^{(g)}{}_{mn} = 0\,, \qquad R^{(g)} = 0\,. \tag{3.15}$$

This simplifies (3.12) and (3.14). Furthermore we see from (3.12) and (3.14) that one possible ansatz for A and B could be that they are logarithmic functions of time t. In other words, we will consider the following ansätze for A and B:

$$A(t) = \alpha \log(t - t_o)\,, \qquad B(t) = \beta \log(t - t_o) \tag{3.16}$$

along with the background G-flux G_{0mnp} as in (3.6) but with $c_1 = 0$. This means that the warp factors are $e^A = (t - t_o)^\alpha, e^B = (t - t_o)^\beta$. These are approximations to the general series solutions (3.1) valid for large $t \gg t_o$, with α, β here playing the role of $n_{\rm max}, m_{\rm max}$ of (3.1) (note that in general, one could imagine the existence of cosmological solutions to the system of higher derivative term equations, represented by the complete t-series (3.1) with nontrivial coefficients).

In the analysis above, observe that we have not ignored the quantum corrections completely. The only constraint we are putting on the unwarped metric will be (3.11). The scalings of the G-fluxes are now important. The component G_{mnpq} is time independent, but could depend on the fourfold coordinates y^m. In this example, it is in fact a vanishingly small constant number. On the other hand G_{0mnpq} is a function of time t via the warp factors A and B. From the form of G_{0mnp} in (3.6), we see that it involves an integral over $D_p\gamma$ and therefore would change the t-dependences. A generic ansatz for the G_{0mnp} component should then be

$$G_{0mnp} = \Big[f(y)\, e^{aA+bB}\Big]_{0mnp}\,, \tag{3.17}$$

where a, b are constants that we will soon determine; and $f(y)$ is a function on the fourfold.

Plugging (3.16) and (3.17) in the first equation (3.12), and taking the limit where $G_{mnpq} \to 0$, we immediately get a relation between α and β. This relation comes from comparing the powers of $(t - t_o)$ on both sides of the equation. The relation is

$$(3 - b)\beta - a\alpha = 1\,. \tag{3.18}$$

In fact, it is easy to see that the same relation would come from both the equations (3.12) and (3.14) as the powers of the exponential are the same in all the equations. When there is a non-trivial G_{mnpq} component the above scaling will not work.

Having gotten a relation between α and β, we now use the two equations, (3.12) and (3.14), to determine these values. Observe that due to the presence of $|G|^2$ in all the equations, there will be another unknown $f(y)$ from the choice of G_{0mnp} in (3.6) (recall that the quantum correction $c_1 = 0$). The relations now are:

$$\alpha^2 + 28\beta^2 + 16\alpha\beta = \frac{f^2}{24} + \mathcal{O}(\epsilon^2)$$

$$\alpha^2 - 8\beta^2 + 16\alpha\beta + \alpha + 8\beta = \frac{f^2}{12}\,. \tag{3.19}$$

These two are simple algebraic equations and therefore the solutions can be easily determined. Combining (3.19) with (3.18), the solution set is given by

$$\left(\frac{k\beta - 1}{a}, \beta\right), \qquad k = 3 - b\,, \tag{3.20}$$

with β being the root of a quadratic equation. The explicit value of β would be

$$\beta = -\frac{1}{2(1+a)}\left[2(k+8a) + (8+ka) \mp \sqrt{(8+ka)^2 + 32(k+8a)(1-a^2)}\right], \qquad a \neq -1\,. \tag{3.21}$$

On the other hand, when $a = -1$, the solution set is

$$(1,0)\,, \qquad \left(\frac{1}{a}\cdot\frac{8}{k-8}, \frac{1}{k-8}\right). \tag{3.22}$$

One can ignore the fractional solution as it gives imaginary values for the background fluxes. We can now plug in (3.22) in (3.19) to get $f = \pm\sqrt{24}$. This way the background G-fluxes G_{0mnp} and the warp factors seem to be completely determined.

However, this is not enough. We still have another equation to go. This is (2.67). For a Ricci flat manifold, this equation simplifies to

$$\begin{aligned}(7\ddot{B} + 2\ddot{A} + 28\dot{B}^2 + 7\dot{A}\dot{B} + \dot{A}^2)g_{mn} = \; & \frac{e^{-6B}}{4}\left(G_{0mpq}G_n{}^{0pq} - \frac{g_{mn}}{6}|G|^2\right) - \\ & -\frac{\kappa^2 T_2 e^{-6B}}{\sqrt{-g}}\left[b_1 e^{-A-4B}\frac{\partial E_q}{\partial(g_{mn}e^{2B})} + g_{mp}\, g_{nq}\dot{y}^p\dot{y}^q \times \right. \\ & \left. \times \left(1 + \frac{1}{2}e^{2(B-A)}|\dot{y}|^2\right)\delta^8(y - y(t))\right].\end{aligned} \tag{3.23}$$

From this, we see that we cannot quite ignore the quantum terms if the manifold is compact.[28] We might be able to ignore the contributions from the membrane terms that depend on powers of $\dot{y}^m$. This equation therefore relates the unwarped metric with the membrane velocities and the quantum terms.

We now face the following puzzle. Once we cannot ignore the quantum terms in (2.67), then taking the trace of this equation relates the warp factors to the membrane and the quantum terms. Furthermore, the membrane and the quantum terms come with different powers of the warp factors. This would mean that the time parameter t does not scale out of the equations completely. The equation (3.23) will also have contributions from G_{mnpq} terms that are typically of order $\frac{e^{2A-8B}}{12}\,\mathcal{O}(\epsilon^2)$. Thus the only possible way to get a solution here would be to consider an essentially non-compact manifold (so that ignoring

[28]If we ignore the quantum terms, then the only solution to all the set of equations (3.12), (3.14), (3.23) and (2.55) is $A = B = G_{0mnp} = 0$ when the internal manifold is compact. This is of course the no-go theorem for warped compactifications [5]. As is well known including higher order corrections or taking a non-compact fourfold invalidates the premises that went into the no-go theorem.

quantum terms do not cause problems with the no-go theorems) and a very slow velocity of the membrane (so that the velocity dependent contributions of the membrane is small). This is, in fact, the precise content of the slow-roll condition (3.9) that we proposed earlier! Therefore the solution set is given by:

$$(\alpha,\, \beta,\, a,\, b,\, k,\, f) = (1,\, 0,\, -1,\, -2,\, 5,\, \pm\sqrt{24})\,. \tag{3.24}$$

This solution set will determine our background metric and the G-fluxes completely, with the assumptions that the membrane motion is very slow (3.9) and the quantum corrections are small. With these conditions, we solve all the equations of motion to get our final result as:

$$\begin{aligned} ds^2 &= (t-t_o)^2\left(-dx_o^2 + dx_1^2 + dx_2^2\right) + g_{mn}\, dy^m dy^n \\ G^{0mnp} &= \pm\frac{\sqrt{24}}{t-t_o}\,, \qquad C_{012} = (t-t_o)^3\,, \qquad G_{mnpq} = [\pm\epsilon]_{mnpq}\,. \end{aligned} \tag{3.25}$$

with $\epsilon \to 0$ for an essentially non-compact fourfold. Once we know the velocities of the membrane from (3.23), we can use (2.71) to determine the values of G_{mnpq}. Knowing the background G-fluxes, (2.55) will tell us how many membranes we have to put in to saturate the anomaly equation. In this way we can determine the complete solution for the system.

The above solution is very interesting in the sense that the internal fourfold is completely time independent![29] This would mean that any dynamics that we get in 2+1 dimensional space from the fourfold will in fact be independent of time. While this time-independence of the compact space can of course be potentially lifted once we move away from the approximations here, it is encouraging to see that the toy model that we study here comes close to something we are interested in. We also repeat here that the regime of validity of this solution is for $t \gg t_o$, as in the general discussion at the end of section 2.1 on potential singularities in these effective gravity solutions.

The question now is whether in type-IIB it is also possible to keep the six dimensional internal space independent of time. The type-IIB metric (2.48) immediately tells us that the internal space will indeed also be time independent. Both the dilaton and the axion for this background vanish locally, as we have shifted any kind of singularities far away. The G-fluxes can allow NS fluxes in type-IIB theory,[30] and this would break supersymmetry.

[29] This however *doesn't* mean that the volume of the fourfold is stabilized! Volume stabilization here will be related to the equivalent problem that we face for the supersymmetric case, namely: the Dine-Seiberg runaway [62]. Therefore having gotten a time independent internal fourfold, our next exercise would be to fix this volume against any runaway by non-perturbative effects. Details on this will be presented in the sequel of this paper.

[30] To see this explicitly, take the background C-fields in (3.5) to be oriented along $C_{mn,11}$, where $x^{m,n}$ lie on the four-dimensional subspace of the six-dimensional base of the fourfold. If the six dimensional base is $K3 \times \mathbb{P}^1$ then $x^{m,n}$ are the coordinates of $K3$. The $C_{mn,11}$ field will then become a B^{NS}_{mn} field in type-IIB. Eq. (3.5), when $\epsilon \to 0$, then gives a time dependent non-commutativity on the seven-brane world volume that wraps the four-dimensional internal space. Such time-dependent non-commutativity has been addressed earlier in [93, 94] in a different context. For our case, the situation is more involved than a simple time dependent non-commutativity because of the y^m dependent in part on B^{NS} which, although small, has the effect of breaking supersymmetry.

The type-IIB metric has the following explicit form:

$$ds^2 = (t - t_o)^2 \sqrt{g_{aa}} \left(-dx_0^2 + dx_1^2 + dx_2^2\right) + \frac{dx_3^2}{g_{33}\sqrt{g_{aa}}} + g_{mn}\, dy^m dy^n \tag{3.26}$$

where we used (2.48) to write the type-IIB metric. The range of the indices $m, n, \ldots$ is $m, n = 4 \ldots 9$, and we have absorbed all numerical constants in the definition of the coordinates. The above metric is not really a de-Sitter like solution that we are ultimately interested in, but as we have made many simplifying assumptions, we really do not expect the metric to look like a de-Sitter one. Most importantly, we see that the x^3 direction does not have any time dependent warp factor because $B = 0$. Therefore, the $x^{0,1,2}$ directions expand differently than x^3 direction (which doesn't expand at all). In fact, the x^3 direction is a small circle with vanishing radius, and therefore the type-IIB theory is effectively a 2+1 dimensional space as we are in the large radius limit of M-theory (3.9) (plus the internal six dimensional space).

Now that we have the M-theory background with fluxes (and also the type-IIB background), it is time to study the motion of the M2 brane. We have already assumed a slow roll condition for the membrane, (3.9). The precise equation of motion for the membrane will be

$$\frac{d^2 y^p}{dt^2} + \gamma^{00} \dot{y}^m \dot{y}^n\, \Gamma^p_{mn} - \frac{1}{2}\, e^{-2B} \dot{y}^m\, G^p{}_{m12} = 0\,, \tag{3.27}$$

where Γ^p_{mn} are the Christoffel symbols measured with respect to the *warped* metric. From the choice of the background, (3.25), $G^p{}_{m12} = 0$ and therefore the acceleration of the membrane can be determined from the velocities alone.

Therefore to summarize: in this example we took very small internal G-fluxes G_{mnpq} on an essentially non-compact fourfold with a slowly moving membrane, so that the quantum corrections could be ignored without violating the no-go theorem. This essentially means that the slow-roll condition has in a sense naturally imposed hierarchies on the G-flux components (which translates to effective non-compactness of the underlying fourfold), ensuring that nontrivial cosmological solutions valid classically at leading order in some of the G-fluxes do not violate equations such as the G-flux anomaly equation. Similar strategies will be seen to apply in the type-IIB discussion of Example 3 in section 3.3. It is conceivable that other nontrivial cosmological solutions exist to the full system of higher derivative equations when the G-flux components are of comparable strength: finding such solutions is potentially harder however, since there is no obvious way to approximate them by a leading classical solution.

3.2 Example 2: arbitrary background fluxes

In deriving the above background, we made many assumptions. Although these are not inconsistent, it would be nice to see if one can have an ansatz for the background that could (a) allow non-trivial function for G_{mnpq} instead of the small value that we took earlier, (b) allow warp factors to depend on the fourfold coordinates y^m also in addition to time t, and

(c) allow membrane and quantum corrections. These are many requirements, and it turns out that this is only possible if we consider the following conditions:

- Since the G_{mnpq} has to be time-independent, the C_{mnp} field will have to follow the ansatze given in (3.3). This means that there is a separate t dependent part $\psi(t)$, and a separate y^m dependent part $\lambda(y)$.
- Since G_{0mnp} has to satisfy (2.50), this component of the flux will in general have to be time dependent. From (3.3), this will be determined by $\dot{\psi}(t)$.
- The warp factor B has to be time independent. This is obvious from the warp factor equation (2.55) and (2.59) as the r.h.s. of (2.55) is time independent. On the other hand, A could depend on time t. The dependence of time t on the warp factor can again be inferred from (2.59): $A(y,t)$ should be a linear combination of some function of time t and some other function of fourfold coordinates.
- For the equations to make sense, *all* the time dependences should scale out. This also tells us that one possible time dependence of the warp factor A could be a logarithmic function of time. This way there is a possibility of scaling out t dependences from all the equations.

It turns out that there is a simple ansatz that takes care of all the points raised above. The ansatze can be written in terms of functions of the internal fourfold that have to be determined, and is given by:

$$\begin{aligned}
\text{Warp factor}: &\quad A = f_1(y) - (1+\epsilon)\log(t-t_o)\,, \qquad B = f_2(y)\,, \qquad \epsilon \to 0\\
\text{G-flux}: &\quad G_{0mnp} = -\frac{c_o}{t-t_o}\,, \qquad G_{mnpq} = h_{mnpq}(y)\,, \qquad G_{012m} = \frac{\partial_m e^{3f_1}}{(t-t_o)^3}\\
\text{C-field}: &\quad C_{mnp} = -c_o\log(t-t_o) + c_{mnp}(y)\,, \qquad C_{012} = \frac{e^{3f_1}}{(t-t_o)^3}\,, \qquad h = dc\\
\text{Membrane velocity}: &\quad |\dot{y}| = \sqrt{\dot{y}^m\dot{y}^n g_{mn}} = \frac{\alpha}{t-t_o}\,, \qquad v = \alpha\, e^{f_2-f_1}
\end{aligned} \tag{3.28}$$

where the tensor indices in the constant quantity c_o is implied, and $f_1(y), f_2(y)$ are scalar functions whereas h_{mnpq} and c_{mnp} are tensors on the fourfold. The quantity α appearing in the velocity is a constant number, and v is the warped velocity defined earlier. Observe also that we have given a very small shift to the time dependence for the warp factor A. The shift is very small but *not* zero. The reason for this will be clear soon.

To compare this solution to what we presented earlier, observe that the C_{mnp} field takes the expected ansatz that we presented earlier in (3.3). The tensor $\lambda_{mnp}(y)$ should be identified with $c_{mnp}(y)$ now. This should now be a generic function of y^m and is not constrained to be small. Furthermore G_{mnpq} is quantized as in (3.4) as before.

Its also clear that the choice of function $f(y)$ in (3.17) cannot be arbitrary. The function $f(y)$ has to be chosen in such a way that

$$\frac{1}{f_2}\left(\log\frac{c_o}{f} - f_1\right) = \mathbb{Z}\,, \qquad \text{and} \qquad a = 1\,, \tag{3.29}$$

where the integer $\mathbb{Z}$ specify the value of b in (3.17). This ambiguity in defining b is reflected in the simple choice of warp factors that we made earlier, namely: the warp factors to depend only on time t. The correct ansatze for the background therefore should be (3.28).

For the moving membrane, the velocity involves a constant α. Our slow roll condition corresponds to α being small. As expected, the world volume metric $\gamma_{\mu\nu}$ is no longer time independent. The metric changes with time in the following way:

$$\gamma_{\mu\nu} = \frac{1}{(t-t_o)^2}\begin{pmatrix} \alpha^2 e^{2f_2} - e^{2f_1} & 0 & 0 \\ 0 & e^{2f_1} & 0 \\ 0 & 0 & e^{2f_1} \end{pmatrix} \tag{3.30}$$

and therefore picks up different values at different points on the fourfold. The total velocity is given by (3.28), and the individual components will be determined later.

Notice also the fact that the ansatz (3.28) does not require us to keep a Ricci flat manifold. In fact it is easy to see that − if we denote the temporal part of $A(y,t)$ by A_o − all equations have a scale factor of e^{2A_o} for every term. This way we can scale away the $(t-t_o)$ dependent parts completely. Therefore the equations would relate only the y^m dependent parts, which we could use to solve for them. However, we will soon see that solutions to these equations exist *if and only if* we incorporate higher derivative terms. In the absence of higher derivative terms, the background goes back to the standard supersymmetric solution.

With this in mind, let us study the equations carefully now. We will typically take $\epsilon \to 0$ in (3.28). The necessity of non-zero ϵ will arise for some of the cases that we shall illustrate soon. First let us us consider the tensor G^{0mnq}. From (3.28), this is determined by c_o. The non-primitivity can now be expressed via c_o and the warp factors in the following way:

$$\begin{aligned} &(1)\quad D_p\left[h^{mnpq} - (*h)^{mnpq}\right] = \left[2c_o e^{2(f_2-f_1)}\right]^{mnq} \\ &(2)\quad D_m\left[e^{3f_1} D_p\left(h^{mnpq} - *h^{mnpq}\right)\right] = 0 \end{aligned} \tag{3.31}$$

which would be easily determined by knowing the warp factors f_1, f_2 and the tensor h^{mnpq}. Indeed, there is a relation that connects them. This is the anomaly equation that combines the warp factors and the h-fluxes as:

$$3\, D_m(e^{6f_2}\partial^m f_1) = \frac{1}{2\cdot 4!}\; h_{mnpq}(*h)^{mnpq} + \frac{2\kappa^2 T_2}{\sqrt{-g}}\left[\delta^8(y-Y) + \frac{X_8}{8!}\right]. \tag{3.32}$$

For a compact fourfold, this will give the anomaly relation connection the h-fluxes with the number of membrane and Euler characteristics.

The next relation would be the Einstein equation (2.61). The t terms of course filter out of this equation, and therefore this will connect the warp factors with the membrane and the quantum terms, $T_m^{(1)}$ and T_q respectively. The main problem, however, comes when we analyse the next equation i.e (2.65). Plugging in the ansatz (3.28) with non-zero ϵ, we get the following relation:

$$\frac{\alpha^2\kappa^2 T_2 e^{-6f_2}}{\sqrt{1-\alpha^2 e^{2(f_2-f_1)}}} + \frac{1}{12}\int d^8y\sqrt{-g}\;|c_o|^2 e^{-6f_2} + \mathcal{O}(G^n R^m) = \epsilon \mathrm{V}_8\,, \tag{3.33}$$

where we have added higher derivative terms. Observe now that (a) in the absence of these higher derivative terms, or (b) when the quantity $\epsilon = 0$, the l.h.s. of (3.33) is a sum of positive definite quantities and would therefore vanish! This means the membrane velocity vanishes ($\alpha = 0$), and $G_{0mnp} = 0$, resulting in a standard supersymmetric compactification with no de-Sitter like solution. However incorporating higher derivative terms this will not be the case. Thus we have our first non-trivial conclusion: *only in the presence of higher derivative terms* the set of equations would have solutions with time dependences.

The existence of these higher derivative terms has been argued in the literature starting with [95]. These are typically of the form of $\int \sqrt{-g}\ [c_{mn} G^m R^n + d_{mn} (\partial G)^m R^n]$, with c_{mn}, d_{mn} being various combinatorial factors that come with different signs. The significance of these terms were realized recently in [93], where an M-theory lagrangian was used to derive the full action for a 6+1 dimensional non-commutative theory. It was observed that the existence of these higher derivative terms are absolutely *necessary* to make sense of the Seiberg-Witten (SW) map [96] for the non-commutative theory. In [93], a few terms of the SW map were derived from the low energy effective action. It was conjectured there that the full M-theory lagrangian with all the higher derivative terms would give the complete SW map from M-theory. Once the higher derivative terms are known, one could easily solve (3.33) to get a relation between membrane velocity, G-fluxes and the warp factors f_1 and f_2.

It turns out that the velocity components of the membrane α^m can be separately determined from (2.70). This is given in terms of h and c_o, the tensors appearing in G_{mnpq} and G_{0mnq}, respectively, in the ansatz (3.28). The velocity components are

$$\alpha^m = \frac{1}{12\kappa^2 T_2} \int d^8y \sqrt{-g} (*h)^{mnpq} (c_o)_{npq} \,, \tag{3.34}$$

and then (2.69) can relate the total velocity α with the warp factors and G-fluxes.

Since we have not used the restriction of Ricci flatness, we can use other equations to determine the Ricci scalar and the Einstein tensor $R^{(g)}$ and $G^{(g)}_{mn}$, respectively. This will provide the complete set of equations for our background. The final M-theory metric will take the following form:

$$ds^2 = \frac{e^{2f_1}}{t^2} \left(-dt^2 + dx_1^2 + dx_2^2\right) + e^{2f_2}\ g_{mn}\, dy^m dy^n \,, \tag{3.35}$$

where we have put $t_o = 0$ for simplicity. The G-fluxes and the membrane will follow the form given in (3.28) when we plug in the values of f_1, f_2, α, h and c_o in those equations.

Finally, observe that the ansatz (3.28) can be put in the general form $A(y,t) = -2B(y) + g(t)$ if we put the condition $f_1(y) = -2f_2(y)$ on the warp factors. This special relation between the f_i is true for the time independent case. For the time dependent case, one can argue that $f_1 + 2f_2 = k$, where k is a covariantly constant function on the fourfold. Putting $k = 0$ will give the required relation between the f_i, and this will simplify all the background equations quite a bit as we saw earlier.

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Therefore to summarize: with arbitrary background fluxes, we can scale out the time dependence of the warp factor A from all the equations to get a fourfold that has time independent warp factor. It is of course an interesting question to ask whether higher derivative terms could spoil this property. In the absence of any concrete calculation, we can only say that as long as the curvatures and the G-fluxes are reasonably small, all these corrections could possible be made small. This way (3.28) can still survive as a valid solution to the system.

3.3 Example 3: towards type-IIB cosmology

In the previous examples we have considered warp factors such that B is time independent, whereas A could be both time and space dependent. This helped us to get some interesting M-theory cosmological solutions. The corresponding type-IIB solution, on the other hand, had time dependent warp factors only along the $x^{0,1,2}$ directions, whereas the x^3 direction was time independent. It could mean that we might need more than two warp factors to get a reasonable type-IIB cosmological solution. We will discuss this case later. In this example, we will try to see how far we can go to get a type-IIB cosmological solution with only two warp factors. In the process, we will find some interesting questions raised on the interpretation and validity of these solutions, that we then discuss briefly.

To start with, first consider a model whose unwarped metric g_{mn} is *not* Ricci flat. This would immediately put some constraints on the warp factors A and B. Our equations of motion are of course (2.57) and (2.64) (or their equivalent simplified form (2.61) and (2.65)) with the warp factors given by the ansatz (3.16). Since the Ricci scalar and Ricci tensors of the unwarped metric are time independent, (2.57) and (2.64) will hold if and only if the time dependences in both the equations can be scaled away. The Ricci scalar $R^{(g)}$ in (2.57) and (2.61) comes with the coefficient $e^{2(A-B)}$. On the other hand, the time derivatives of the warp factors all go as t^{-2} if we use (3.16) in (2.57) or (2.64). This gives us the first non-trivial relation between the warp factors:

$$\alpha - \beta = -1\,. \tag{3.36}$$

To get the other equation we need to see how G_{0mnp} or G_{mnpq} scale with respect to the warp factors. Assume now that G_{mnpq} now satisfies the equation (2.52) with some tensor γ_{mnpq} that is vanishingly small. From (2.50) we see that G_{0mnp} will continue to satisfy (3.6) and therefore the G-fluxes for our case will be:

$$G_{012m} = \partial_m e^{3A}\,, \qquad G^{0mnpq} = e^{-A-2B}\, c_o\,, \qquad G_{mnpq} = e^{-3A}\gamma_{mnpq}\, c_2\,, \tag{3.37}$$

with c_o, c_2 as some undetermined constants. Knowing the background G-fluxes as some functions of the warp factors, we can use (2.61) or (2.65) to get the next non-trivial relation between the warp factors:

$$\alpha = -2\beta \tag{3.38}$$

where we compared the powers of the Ricci scalar and the $|G|^2$ terms in (2.57) or (2.64). We see that the relation (3.38) is strongly reminiscent of the second relation of (2.47)! Although (2.47) is more complicated because of its dependence on space coordinates y^m, the temporal part of this equation does satisfy $A = -2B$. Furthermore, having $A = -2B$ simplifies all the Einstein equations considerably, as we saw in detail in the previous sections. Now we see another advantage of having $A = -2B$: the scalings of the Ricci tensors (or the curvature scalar) can be extracted from the equations and therefore we may not impose the condition of Ricci flatness for our case.

The above analysis unfortunately leads to a problem when we encounter the anomaly equation (2.55). Since the G_{mnpq} tensor has become time dependent the anomaly equation will no longer hold because the membrane and the X_8 terms are in general time independent. One way out of this is to completely ignore these terms. In other words, we may not consider any branes (D3 or D7 branes) in the type-IIB scenario,[31] i.e. we could only study the supergravity background without any brane inflation. Alternatively, (2.55) can receive corrections such that the time dependences of $\Box e^{6B}$ can be cancelled by the flux term.

In any case, this is of course not a serious problem as generic study of cosmological solutions have been done before without invoking brane inflation *per-se*. Without requiring inflation, our system becomes perhaps less interesting, but nevertheless much simpler, although one might wonder if in the absence of the X_8 term, there could be any solution at all. For a *compact* internal fourfold there is *no* solution. But when we make the manifold non-compact and keep the J_o, E_8 and higher derivative terms, then solutions would exist.

Solving for α and β we get $\alpha = -\frac{2}{3}$ and $\beta = \frac{1}{3}$. The type-IIB metric can now have an overall factor in front of $x^{0,1,2,3}$ because of its resemblance with the second equation of (2.47). In fact the explicit type-IIB metric can be easily shown using (2.48) to be:

$$ds^2 = \frac{1}{t}\left[\sqrt{g_{11,11}}(-dt^2 + dx_1^2 + dx_2^2) + \frac{dx_3^2}{g_{33}\sqrt{g_{11,11}}}\right] + t\, g_{mn}\, dy^m\, dy^n\,, \tag{3.39}$$

with $m, n = 4 \dots 9$ and the other variables are already defined in (2.48). To glean four dimensional physics here, note that this metric is of the form (2.15) with a time-dependent warping to the six dimensional space: a naive dimensional reduction would yield a time-dependent 4D Planck constant. Thus one needs to first absorb this time-dependence, going to the 4D Einstein frame as discussed in general in (2.15)–(2.18). From there, we read off $p(t) \sim \frac{1}{t}$, $h(t) \sim t$ giving $\eta^2(t) = h^{6/2}p \sim t^2$ as the proper scale factor of this cosmology. Using (2.1)–(2.4), this gives the FRW scale factor to be $a(\tau) \sim \tau^{1/2}$, which is precisely a four dimensional radiation-dominated cosmology.[32] Clearly this effective solution breaks down at $\tau \sim 0$ where we necessarily must resort to the full string theoretic description, as we have discussed in Example 1 (section 3.1), and more generally at the end of section 2.1.

We will elaborate in more detail below, but observe here that there are non-trivial functions of y^m that are distributed in an uneven way among $x^{1,2}$ and x^3 which spoils required properties of the metric because we took $A = -2B$ instead of the second equation of (2.47). Therefore, first we have to search for a metric that can get rid of these problems.

[31] Recall that the singularities of X_8 are the points where seven-branes are located.

[32] Recall, as we have mentioned in section 2.1, the expressions $a(T) \sim T^{[2/3(1+w)]}$ for the time-scaling of the scale factor corresponding to a perfect-fluid with equation of state $p = w\rho$.

To find an ansatz for the warp factors, we have to consider a relation between the warp factors that is of the form $A = -2B + g(y)$. With this relation, it will no longer be possible to keep B independent of time t. As discussed above, this is only possible (at least for this scenario) when we ignore the membrane and the seven-brane singularities. This means that the internal manifold is essentially non-compact. The ansatz now will be the following:

$$\begin{aligned}
\text{Warp factor :} \qquad A &= -2b\log(t-t_o) + 2g_1(y) + g(y)\,, \qquad & B &= b\log(t-t_o) + g_1(y) \\
\text{G-flux :} \quad G_{mnpq} &= g_3(y)(t-t_o)^{3b}\,, \qquad & G_{0mnq} &= \frac{g_4(y)}{(t-t_o)^{1-3b}}\,, \\
G_{012m} &= \frac{\partial_m[e^{3g(y)+6g_1(y)}]}{(t-t_o)^{6b}}
\end{aligned} \tag{3.40}$$

where the tensor indices of g_3, g_4 are implied. Non-trivial solutions for g_i exist only when we incorporate higher derivative terms, as can be seen by plugging this ansatz in the set of equations of section 3.2. We will not present the full solution set for the y^m dependent function in this paper. It will suffice to give the value of b and the form of the C_{mnp} field. This is given by

$$b = \frac{1}{3}\,, \qquad C_{mnp} = [(t-t_o)g_5(y)]_{mnp} \tag{3.41}$$

with $g_5(y)$ being yet another function that one could easily relate to the other $g_i(y)$. The value of b will tell us the overall conformal factor. We see that this is the same as we got earlier with a rather restrictive choice of G-fluxes. The final metric will now only have an overall conformal factor, thanks to (2.47), and is given by:

$$ds^2 = c_4 t^{-1}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + t\ \tilde{g}_{mn}\, dy^m dy^n \tag{3.42}$$

where $c_4 = \frac{e^{-3g_1}}{g_{33}\sqrt{g_{11,11}}}$ and $\tilde{g}_{mn} = e^{3g_1}\sqrt{g_{11,11}}\ g_{mn}$. As we have seen, this is a radiation-dominated cosmology. It is also easy to see from (3.41) and (3.40) that G_{mnpq} will become time dependent whereas G_{0mnp} will be time independent. All of them will of course now be non-trivial functions of the internal space coordinates.

While the above solution seems quite a benign 4D cosmology, it is important to note that this solution is effectively valid only in the noncompact limit of the fourfold as we have discussed above. Thus the validity of a four-dimensional description is not particularly clear: essentially this is the description on *some* hyperslice in the noncompact higher dimensional spacetime. We expect that if we can find an embedding of such a solution in a compact space, these solutions will get modified to possibly different four dimensional cosmologies.

In the context of a cosmological solution in a compact setting, it is interesting to ask if there are general restrictions imposed on the space of cosmologies by such an M-theory lift involving higher derivative terms (which are necessarily nonzero in the compact case), and when the higher derivative terms can be treated as giving rise to a sensible effective fluid-like matter field configuration. In this case, it would be interesting to study how the energy contributions from the higher derivative terms intertwine with known energy conditions and cosmological bounds in four dimensions.

3.4 Example 4: three warp factors

Our next example is a model with a different choice of the warp factors for the T^2 fiber directions, i.e., we consider now a metric of the fourfold and the spacetime of the form

$$ds^2 = e^{2A}(-dx_0^2 + dx_i dx_i) + e^{2B} g_{mn}\, dy^m dy^n + e^{-A}(dx_3^2 + dx_{11}^2) \tag{3.43}$$

where x^3, x^{11} denote the directions of the T^2 fiber and $i = 1, 2$. In terms of our earlier metric (2.33), this is the case when $C = -\frac{A}{2}$, along with a complex structure of $\tau = i$ for the fiber torus for simplicity. Now let us consider the following generic choices of the warp factors A and B:

$$A(y,t) = \frac{2}{3}\log f(y)g(t), \qquad\qquad B(y,t) = \frac{1}{2}\log h(y) f^{\frac{1}{3}}(y) g^{\frac{1}{3}}(t)\,. \tag{3.44}$$

At this point, such a choice of warp factors may look completely arbitrary. But once we go to type-IIB we see that the metric takes the following form:

$$ds^2 = f(y)g(t)\left(-dx_0^2 + \sum_{i=1}^{3} dx_i dx_i\right) + h(y)\; g_{mn}\, dy^m dy^n \tag{3.45}$$

which is in fact the kind of metric that we are looking for. But this is not all. If we further impose the following restriction on h and f:

$$h(y) = f^{-1}(y) \equiv \sqrt{j(y)} \tag{3.46}$$

then the type-IIB metric will take the standard form of a D3-brane metric with a harmonic function $j(y)$, at least at some initial time t_o when $g(t_o) \approx 1$. For any other time t, $g(t)$ could be any arbitrary value. However if we want to mimic a de-Sitter background then we can fix some particular value of $g(t)$ in terms of a cosmological constant Λ as

$$g(t) = \frac{1}{\Lambda t^2}\,. \tag{3.47}$$

If we switch on a non-primitive G-flux G_{mnpq} on the six dimensional base of the fourfold, then the cosmological constant can be explicitly determined in terms of G_{mnpq} as

$$\Lambda = -\frac{1}{12 j^2}\left[\Box j + \frac{2}{3}|G|^2\right] + \cdots \tag{3.48}$$

where $|G|^2 = G_{mnpq}G^{mnpq}$ and the dotted terms involve contributions from the membrane and the quantum terms whch would eventually make $\Lambda > 0$. Taking the G-fluxes to have legs along the T^2 fiber direction will in fact make the analysis much more involved, as one can show that the background Einstein equations become time dependent for such a choice of fluxes.

Of course the choice of warp factors that we took here has no *a-priori* justification. For a more generic discussion we have to start with a metric that has three distinct warp factors in M-theory as in (2.33) and also with a non-trivial complex structure for the fiber torus.

In fact it is easy to generalize (3.45) to incorporate the information about the complex structure. Namely, from (2.45), we have to take

$$f(y)g(t) \to f(y)g(t)\,|\tau|\,\tau_2^{-1}\,, \qquad h(y) \to h(y)\,|\tau|\,\tau_2^{-1} \tag{3.49}$$

when we choose the complex structure as $\tau = \tau_1 + i\,\tau_2$.

In a background with three warp factors A, B and C one has the freedom to keep the internal manifold time independent, but the spacetime with some requisite time dependence. However, from (2.47) we see that the analysis of the equations of motion that we did here will have to change, as the scalings etc have changed. Also it is not clear whether the ansatz for G-flux (2.21) will remain the same with three warp factors. But there are some issues we can study without going to the G-flux ansatz (2.21):

- First, we observed for the two warp factor case that the time and space dependences of the warp factors isolate. In fact keeping B only as a function of y^m gave us a type-IIB six manifold with no time dependence. On the other hand keeping B as $B(y)$ we were unable to reproduce the kind of metric that we wanted in 3+1 dimensions. It is interesting to observe that the three warp factor case can give us a 3+1 dimensional space with an overall time-dependent warp factor and a six dimensional internal manifold with a warp factor that is only a function of y^m, if we consider the following ansatz for A, B and C:

$$A(y,t) = f_1(y) + g(t)\,, \qquad B(y,t) = f_2(y) + \frac{1}{4}g(t)\,, \qquad C(y,t) = f_3(y) - \frac{1}{2}g(t) \tag{3.50}$$

  where $f_i(y)$ are some functions on the internal fourfold. Whether the background equations of motion allow this ansatz still remains to be seen.

- The second point is the analysis of (3.33), where we showed that when the r.h.s. of the equation vanishes, this equation will have solution iff we put in higher derivative terms in G-fluxes. When we have three warp factors, the r.h.s. of (3.33) can get an additional contribution from the warp factor C which could be a function of time (although the warp factor B may not be). Therefore (2.65) is now given by (no sum over i):

$$G_{00} + G_{ii} = \dot{A}^2 - \ddot{A} - 2(\ddot{C} + \dot{C}^2 - 2\dot{A}\dot{C}) - 6(\ddot{B} + \dot{B}^2 - 2\dot{A}\dot{B})\,. \tag{3.51}$$

  Assuming now the above ansatz (3.50) is an allowed one, it is easy to see that (3.51) can be made positive definite if the time dependence of the warp factors $g(t)$ obey the inequality:

$$\dot{g}^2 \,\geq\, \frac{4}{3}\ddot{g} \tag{3.52}$$

  with an equality leading to the same problem that we encountered earlier. It will be interesting to see whether this could now have solutions without assuming higher derivative corrections.

- The third point concerns the membrane velocity. This is related to the Einstein tensor G_{0m}, which in turn is related to non-primitivity. Switching on a time dependent warp factor C, will give us the following contribution

$$G_{0m} = -\frac{3}{2}\dot{g}\,\partial_m \log \tau_2 \tag{3.53}$$

  and therefore change the membrane velocity (see (2.69) for the derivation). A similar analysis can be done for the time dependent B_{NS} in type-IIB theory, which comes from the $C_{mn,11}$ field as discussed earlier. The energy momentum tensor of this field is now correlated with $G_{11,11}$. We can also switch on C_{mn3} giving rise to a $B_{RR}(y,t)$ field in type-IIB theory. These two fields together will break supersymmetry for our case. We would now need a five form field in type-IIB to satisfy equation of motion. This in turn means that we need a G-flux ansatz for G_{012m}.

Therefore to summarize: we see that with three (or more warp factors) we can overcome some of the problems related to the higher derivative terms that plague the two warp factors case. It would seem that, with three warp factors, a de-Sitter type cosmology could be derived as there are much more freedom within the range of variables. A detailed analysis of this is beyond the scope of this paper and left for the sequel to this paper.

3.5 Example 5: a new background for the D3/D7 system

In all the previous examples we have kept the metric g_{mn} of the internal space (in say type-IIB) undetermined. We will now be a bit more explicit and study some possible choices for g_{mn}. In fact we already know one example, namely the case when g_{mn} is the metric for a $K3 \times T^2/\mathbb{Z}_2$ background geometry. This background was chosen in [25] because the metric of the system − at least when $K3$ is at the orbifold limit − is known. Furthermore, there is also a concrete F-theory realization of the system as a $K3 \times K3$ fourfold with G fluxes, that we can exploit to study a system of a $D3$ brane in the background of seven-branes. In the $\mathbb{Z}_2$ *orientifold* limit, we have a system of $D7$ branes and $O7$ planes that are arranged in such a way as to allow a D_4^4 singularity. Away from the orientifold point, we have generic seven-branes, and the cosmological $D3/D7$ system was studied by keeping all the others seven-branes far away. The choice of non-primitive G fluxes were made in such a way that it allowed a single $D3$ brane to move towards the single seven-brane. Furthermore, in this limit the background changed to $K3 \times \mathbb{P}^1$ because of non-perturbative corrections. The $\mathbb{P}^1$ is not smooth, but has singular points where the seven-branes are located.

Although the above compact space is simple enough to give us a nice cosmological scenario, the background suffers from one important drawback: away from the orbifold limit, the metric of K3 is not known, and unless we go to this orbifold limit of K3 we cannot, in practice, evaluate any precise cosmological effects from this scenario. Furthermore, due to the inherent orientifold nature of this background, the polarizations of B_{NS} fields are rather restricted. Overcoming these issues would therefore require a new embedding for the $D3/D7$ system, which would have the following properties:

- Allow $D3$ and $D7$ branes.[33] In other words, should have an explicit F-theory realization as a fourfold with G fluxes. These G-fluxes will give H_{NS} and H_{RR} fields in type-IIB.
- The six dimensional base of the fourfold should have a supergravity description. This is required to understand the motion of the $D3$ brane towards the $D7$ brane.

One of the known six dimensional manifolds that would satisy all the criteria is a deformed conifold. The metric of the deformed conifold is exactly known.[34] What remains, is to include the seven-branes in this background.

To include seven-branes, we require an F-theory fourfold that will have the deformed conifold as its base. The T^2 fiber of the fourfold will have to degenerate at some number of points on the base. The degeneration points are where the seven-branes occur. Let us therefore first construct this fourfold.

The fourfold that we require for our case should be compact, although we will not consider the compactness too seriously for the subsequent analysis. Some analysis of such a fourfold is given in [98]. Taking the usual deformed conifold equation defined in $\mathbf{C}^3$, we can compactify to a projective variety $\mathcal{B}_\mu$ in $\mathbf{P}^4$ by adding $\mathbf{P}^1 \times \mathbf{P}^1$ to the boundary of the deformed conifold, which itself is symplectically isomorphic to the cotangent bundle $T^*\mathbf{S}^3$ over the three sphere. The quadratic equation now becomes:

$$\mathcal{B}_\mu : \; z_0^2 + z_1^2 + z_2^2 + z_3^2 - \mu z_4^2 = 0 \,. \tag{3.54}$$

The quadric threefold $\mathcal{B}_\mu$ does not develop any new singularities at infinity and is thus smooth for $\mu \neq 0$ and has a conifold singularity at $(0,0,0,0,1) \in \mathbf{P}^4$ when $\mu = 0$. Moreover, the anti-canonical bundle $-\mathcal{K}_{\mathcal{B}_\mu} := \wedge^3 \mathbf{T}\mathcal{B}_\mu$ will be the restriction of $\mathcal{O}(3)$ of $\mathbf{P}^4$ to $\mathcal{B}_\mu$ by the adjunction formula, and hence it is very ample. From the Kodaira vanishing theorem, one can also show that $H^i(\mathcal{B}_\mu, \mathcal{O}) = 0$ for $i > 0$. We now define a fourfold Y_μ as a subvariety in the projective bundle $\mathcal{P}(\mathcal{O} \oplus \mathcal{L}^2 \oplus \mathcal{L}^3)$, with $\mathcal{L} := \mathcal{K}_{\mathcal{B}_\mu}^{-1}$. The Weierstrass equation is given by

$$y^2 z_o = x^3 + f z_o^2 x + g z_o^3 \tag{3.55}$$

where z_o, x, y, f, g are the sections of $\mathcal{O}, \mathcal{L}^2, \mathcal{L}^3, \mathcal{L}^4, \mathcal{L}^6$ respectively. Since the anti-canonical bundle $\mathcal{L} = \mathcal{O}(3)|_{\mathcal{B}_\mu}$ is very ample, we may choose f and g so that the fourfold Y_μ is smooth. By the projection formula, one can see that Y_μ is Calabi-Yau since $H^i(\mathcal{B}_\mu, \mathcal{O}) = 0, i > 0$. By construction, the natural projection $\mathcal{P}(\mathcal{O} \oplus \mathcal{L}^2 \oplus \mathcal{L}^3) \to \mathcal{B}_\mu$ induces a fibration $Y_\mu \to \mathcal{B}_\mu$ whose fibers are elliptic curves. F-theory on the Calabi-Yau fourfold Y_μ is by definition

[33] We need a background that allows at least *two* $D7$ branes instead of one. Having two $D7$ branes means that on the $D3$ brane we will see a global symmetry of SU(2) which would, in turn, allow *semi-local strings* instead of cosmic strings. The existence of such semi-local strings in the $D3/D7$ system solves the problem of the otherwise generic overproduction of cosmic strings [55, 30]. Some other aspects of semi-local strings will be discussed later in this paper.

[34] In a somewhat different context, cosmological models involving the conifold transition were recently studied in [97].

type-IIB theory compactified on the base $\mathcal{B}_\mu$ with background axion-dilaton field λ whose j-invariant is given by the usual formula with various (p, q) seven-branes appearing at the loci where the elliptic fibration degenerates. Using the Riemann-Roch theorem for $\mathcal{B}_\mu$ and integrating over the elliptic fibers, one can evaluate the Euler-Characteristic χ of Y_μ to be 19728.

The above construction of the fourfold gives us a way to see the structure of the base. As we can easily see, the base is not quite a deformed conifold as it not possible to have $c_1 = 0$ for the base, where c_1 is the first Chern class. Although the metric can be shown to be approximately a warped deformed conifold. The warp factors will come from the backreactions of the seven-branes, fluxes and the D3-brane. For the specific manifold that we constructed, we require the H_{NS} and the H_{RR} fluxes to satisfy

$$\int_{\mathcal{B}_\mu} H_{NS} \wedge H_{RR} = 822 - n\,, \tag{3.56}$$

where n is the number of $D3$ branes. Normalizing the NS and RR fluxes to satisfy this will be essential to have $D3$ branes in this scenario. For the simplest case we would require $n = 1$, i.e a single $D3$ brane.

But this is not enough. The specific configuration that we require should (a) allow the $D3$ brane to move towards the $D7$ branes and (b) not allow the problematic ANO cosmic strings. This two aspects can be easily solved by switching on H_{NS} and H_{RR} fluxes that are not *primitive*. Once primitivity is gone, the fluxes in the background with the compact base $\mathcal{B}_\mu$ will break supersymmetry and the $D3$ brane will move towards the $D7$ branes.

To get rid of the ANO strings, we have to isolate two $D7$ branes at a point on the compact space and keep all the other seven-branes far away, as discussed in the footnote earlier. This configuration will therefore be similar to the one developed in [30] for the $D3/D7$ system in $K3 \times T^2/\mathbb{Z}_2$ background. The non-primitive fluxes will drive the $D3$ brane towards the two $D7$ branes, and this will be the Coulomb phase for our inflationary model. The story is similar to our earlier papers [25, 30] so we will not repeat it here anymore. Interested readers may want to look up [25] and [30] for details.

What we want to concentrate on in this section is the precise metric for the system. We already know from (3.54) that the F-theory base of our fourfold, $\mathcal{B}_\mu$, has $b_3 \geq 1$, where b_3 is the third Betti number and therefore supports three cycles. On the other hand we have shown above that, by Kodiara's vanishing theorem,

$$H^i(\mathcal{B}_\mu, \mathcal{O}) = 0 \qquad \text{for } i > 0 \tag{3.57}$$

which implies that the fourfold Y_μ is a Calabi-Yau. For our case however we have to change the geometry a little bit by shifting most of the seven-branes far away. What we require is the fourfold base to be approximately a Calabi-Yau, or more appropriately, a deformed conifold. Since the first Chern class of the base $\mathcal{B}_\mu$ is non zero, this might seem difficult to realize in principle. In practice however the situation can be controlled. We can shift the seven-branes in such a way that the metric of the system is approximately a warped deformed conifold. In fact we have to take care of the following things:

- The effect of warping coming from the back reactions of branes and fluxes on the geometry.
- The precise metric of the *deformed* conifold,[35] and
- The background three-form fluxes that are non-primitive, in other words $H_{NS} \neq *H_{RR}$.

In addition to these, we have to make sure that the background satisfies all the string equations of motion. This is now a rather complicated issue. Let us see how far can we proceed towards getting the full answer. First, observe that the background with $D3, D7$ and fluxes on the fourfold base $\mathcal{B}_\mu$ is closely related to the recently studied type-IIB manifolds in the geometric transitions setting of [58].

We could then follow the procedure laid down in [58] to get our background metric. However one immediate problem is that the background studied in [58] is supersymmetric. But what we require is actually a non-supersymmetric (and hence non-static) background. From earlier sections, we know that this can be easily achieved by taking fluxes that are non-primitive. With a time-independent choice of the warp factors for the internal space we can in fact use the full machinery of geometric transition to determine our final answer. Our goal therefore is to determine g_{mn} in

$$ds^2 = e^{A(y,t)}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + e^{B(y)}\; g_{mn}\, dy^m dy^n \tag{3.58}$$

with the assumption that $A(y,t)$ and $B(y)$ to be determined by the analysis that we presented earlier.

Therefore, to start off, first let us use some simple approximations that we used in [57] and [58]. We want to remind the reader that these approximations are just done to simplify the ensuing analysis. The six-dimensional manifold $\mathcal{B}_\mu$ will have coordinates $(r, \phi_1, \theta_1, \phi_2, \theta_2, \psi)$, where (θ_i, ϕ_i) $(i = 1, 2)$ are related to two S^2 metrics (that doesn't always imply that we will have two topologically non-trivial S^2 in our setup). The radial coordinate will be denoted as r, and ψ is the usual U(1) fibration over the S^2 bases. With this, we can now use the same trick that was used in [57] and [58] to get the final type-IIB metric, namely: use the geometric transition from type-IIB to another type-IIB solution. Thus our initial solution will be related to the one in [99, 57, 58] but with seven-branes inserted in. In other words we are choosing the metric:

$$ds^2 = h_6\, dr^2 + (dz + E\, dx + F\, dy)^2 + |dz_1|^2 + |dz_2|^2 \tag{3.59}$$

with dz_i, dz_2 being two-tori with two different complex structures and E, F as some functions given in [57, 58] (see also [99]). The background also had non-trivial dilaton ϕ and

[35] As mentioned earlier and will also be clear soon, our six dimensional manifold is not a Calabi-Yau and only resembles the usual deformed conifold in some local region. Thus by *deformed* conifold we mean a non Kähler deformation of a *compact* Calabi-Yau metric.

axion $\tilde{\phi}$ switched on. The coordinate redefinition that we are using here will be:

$$(dx, dy, dz) \equiv \left(\frac{1}{2}\sqrt{\gamma_1\sqrt{h}}\sin\theta_1\, d\phi_1, \frac{1}{2}\sqrt{\gamma_2\sqrt{h}}\sin\theta_2\, d\phi_2, \frac{r}{2}\sqrt{\gamma_1'\sqrt{h}}\, d\psi\right) \tag{3.60}$$

where h is the overall warp factor that comes from the back reaction of fluxes and branes on the geometry. This coordinate redefinition is made to the geometry *before* the geometric transition (in the language of [58]). Thus the threefold base of F-theory is approximately a resolved conifold with the resolution parameter being given by:

$$\frac{1}{2}\sqrt{\gamma_2 - \gamma_1}\,. \tag{3.61}$$

The physical meaning of x, y and z is already explained in [57]. They are related to the tori that we use in place of the two spheres. In other words, we will have two tori with coordinates (x, θ_1) and (y, θ_2). The coordinate z remains as the usual U(1) fibration over tori bases. Furthermore, these tori will have non-trivial complex structures that can be either integrable or non-integrable. What we require for geometric transitions is non-integrable complex structures, although, as discussed in [57], integrable complex structures lead us very close to the expected right metric in the mirror. We will not elaborate on these here anymore, and the readers may want to see [57] and [58] for more details.

However, there is one subtlety that we have to mention at this stage. This is related to the configuration of seven-branes in our setup. Recall that F-theory generically predicts seven-branes that could be either $D7$ branes and/or generic non-local seven-branes charged under both axion and dilaton of type-IIB theory. The backreactions of the branes and fluxes tell us that the type-IIB metric can be determined by two independent variables E and F from (3.59) as

$$j_{xx} = 1 + E^2\,, \qquad j_{yy} = 1 + F^2 \tag{3.62}$$

with all other components of the metric (3.59), $j_{\mu\nu}$, some functions of E, F and the complex structures only. With these choices of E and F, F-theory predicts that the metric along x, y and θ_1, θ_2 corresponds, in fact, to two different tori with complex coordinates $d\chi_1 \equiv dx + \tau_1\, dy$ and $d\chi_2 \equiv d\theta_1 + \tau_2\, d\theta_2$ and complex structures τ_1 and τ_2 respectively. Observe that these complex structures are different from the original complex structures of the (x, θ_1) and (y, θ_2) tori. The new complex structures that mixes x, y and θ_1, θ_2 are given by [58]

$$\tau_1 = \frac{1}{1+E^2}\left[cEF + i\sqrt{\alpha^{-1} + (1-c^2)E^2F^2}\right], \qquad \tau_2 = i\sqrt{\frac{\gamma_2}{\gamma_1}} \tag{3.63}$$

where α is defined as $\alpha = (1 + E^2 + F^2)^{-1}$.

The value of c in (3.63) is now important. As discussed in [58], the constant c can take only two values, $c = 1$ or $c = 0$. The two values of c distinguish two possible configurations we can have with the seven-branes, namely: *at* the orientifold point or *away* from the orientifold point. At the orientifold point c is zero and therefore the two tori (x, y) and (θ_1, θ_2) are square tori. On the other hand, away from the orientifold point, the value of c is typically 1. In fact this is the situation that we really require for our case, because we have to move the seven-branes far away, keeping only two $D7$ branes at a point. According to the F-theory analysis, this amounts to having the orientifold planes split into generic seven-branes. As a result the (x, y) torus pick up a complex structure that can be determined from (3.63) as

$$\tau_1 = \frac{EF + i\sqrt{1+E^2+F^2}}{1+E^2} \tag{3.64}$$

with the other (θ_1, θ_2) torus remaining a square torus.

At this point, one would like to compare our background to the one that appeared in [58] (at least before the geometric transition). The configuration in [58] corresponds to $c = 0$ and the seven-branes (with orientifold seven planes) located at points on the (x, y) torus. For our case, we require a situation that is the exact opposite of what we had in [58]. We require $c = 1$ with the seven-branes (no more orientifold planes) located at points on the (x, θ_1) torus.

The above story was before the geometric transition in the type-IIB theory. Having set the value of c and the required torus, we can do a geometric transition to get to our final, albeit, static background. The final metric is

$$\begin{aligned} ds^2 =\ & h_1(dz + a_1 dx + a_2 dy)^2 + h_2(dy^2 + d\theta_2^2) + h_4(dx^2 + h_3\, d\theta_1^2) + \\ & + h_5 \sin\psi(dx\, d\theta_2 + dy\, d\theta_1) + h_5\cos\psi(d\theta_1 d\theta_2 - dx\, dy) + h_6\, dr^2\,, \end{aligned} \tag{3.65}$$

where we can now use the variable of the metric (3.59) to determine the values of h_i as follows (see also [58]):

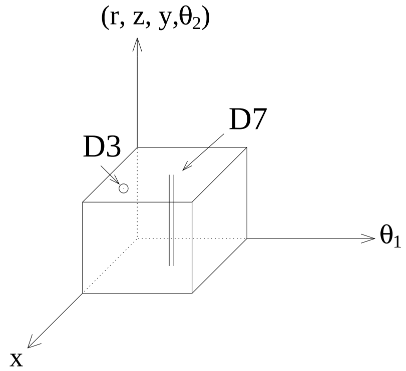
$$\begin{aligned} h_1 &= \frac{e^{-2\phi}}{\alpha_0 CD}\,, \qquad h_2 = \alpha_0(C + e^{2\phi}E^2)\,, \qquad h_4 = \alpha_0(D + e^{2\phi}F^2)\,, \qquad h_5 = 2\alpha_0 e^{2\phi}EF \\ h_3 &= \frac{C - \beta_1^2 E^2}{\alpha_0(D + e^{2\phi}F^2)}\,, \qquad a_1 = -\alpha_0 e^{2\phi}ED\,, \quad a_2 = -\alpha_0 e^{2\phi}FC\,, \qquad \alpha = \frac{1}{1+E^2+F^2} \\ \alpha_0 &= \frac{1}{CD + (CF^2 + DE^2)e^{2\phi}}\,, \qquad \beta_1 = \frac{\sqrt{\alpha_0}e^{2\phi}}{\sqrt{e^{2\phi}CD - (C + e^{2\phi}E^2)(1-D^2)F^{-2}}} \\ C &= \frac{\alpha}{2}\left(1 + F^2 - \frac{EF\sqrt{1+F^2}}{\sqrt{(1+E^2)}}\right), \qquad D = \frac{\alpha}{2}\left(1 + F^2 + \frac{EF\sqrt{1+F^2}}{\sqrt{(1+E^2)}}\right). \end{aligned} \tag{3.66}$$

This way we can determine the metric (3.58) completely, with the only unknowns being the warp factors A and B. We believe that using three warp factors in the M-theory setting, these will be uniquely determined. This would therefore be the static background that would allow seven-branes, $D3$ brane and non-trivial three form fluxes. The axion-dilaton distribution can be determined from the corresponding F-theory curve. A slight deviation from the primitivity will trigger the motion of the $D3$ brane towards the $D7$ branes. Both the $D3$ brane and the $D7$ branes are now located at points on the (x, θ_1) torus. The $D7$ branes therefore wrap directions (y, θ_2, z, r) and are stretched along $x^{0,1,2,3}$. The $D3$ brane is along the $x^{0,1,2,3}$ directions. We can also choose the three forms $H_{NS} \equiv H$ and $H_{RR} \equiv H'$ in the following way:

$$\begin{aligned} H =\ & \partial_r B_{x\theta_1}\, dr\wedge dx\wedge d\theta_1 + \partial_{\theta_2} B_{x\theta_1}\, d\theta_2 \wedge dx\wedge d\theta_1 + \\ & + \partial_r B_{y\theta_2}\, dr\wedge dy\wedge d\theta_2 + \partial_{\theta_1} B_{y\theta_2}\, d\theta_1\wedge dy\wedge d\theta_2 \end{aligned}$$

$$H' = \partial_{\theta_1} B'_{zy}\, dz \wedge d\theta_2 \wedge dy + \partial_{\theta_2} B'_{zy}\, dz \wedge d\theta_1 \wedge dy + \\ + \partial_{\theta_1} B'_{zx}\, dz \wedge d\theta_1 \wedge dx + \partial_{\theta_2} B'_{zx}\, dz \wedge d\theta_2 \wedge dx\,, \tag{3.67}$$

where $B_{x\theta_1}(r, \theta_2)$ and $B_{y\theta_2}(r, \theta_1)$ are the B_{NS} fields and $B'_{zx}(\theta_1, \theta_2)$ and $B'_{zy}(\theta_1, \theta_2)$ are the B_{RR} fields. Observe that these B fields are functions of the radial coordinate r as well as the angular coordinates θ_1 and θ_2. Furthermore, since we do not require primitivity at the Coulomb stage of our model, we can take a simplified choice of the threeforms by putting $B'_{zx} = B'_{zy} = 0$. This would mean that we only have H_{NS} in our background. This choice will be useful to study black holes in this scenario. We will also discuss the changes that can appear when we switch on H_{RR}. In the figure below:



the $D3$ brane is at a point on the (x, θ_1) direction similar to the two $D7$ branes which are stretched along the (r, z, y, θ_2) directions. As discussed above, the fluxes would drive the $D3$ brane towards the two $D7$ branes. The whole system is of course embedded in a deformed conifold background. In the figure above, the two $D7$ branes wrap the three cycle (z, y, θ_2) of the deformed conifold. Furthermore when we switch off H_{RR} to keep only the H_{NS} field, we have to consider multiple $D3$ branes satisfying (3.56). In the above figure we can still have the same configuration, but now we have to keep other $D3$ branes far away (along with the set of seven branes). This way the cosmology of the model will remain intact.

One might ask if there are other interesting geometries that one could use to construct cosmological models. In this context, we recall that [100] studies the physics of closed string tachyons localized to $\mathbb{C}^3/\mathbb{Z}_N$ nonsupersymmetric noncompact orbifold singularities and the phenomena therein using the connections between the worldsheet orbifold conformal field theory, RG flows and the toric geometry of these orbifolds. The geometry in codimension three is more intricate than the lower dimensional cases [101] [102] [103]. In particular if a more relevant tachyon condenses in the process of condensation of a less relevant tachyon, there are flip transitions [104] between topologically distinct tachyonic resolutions of the original singularity (contrast with the more familiar flops in Calabi-Yau spaces, executed by moduli). It is interesting to ask if there are compact embeddings of these $\mathbb{C}^3/\mathbb{Z}_N$ singularities in the presence of nontrivial background fluxes, which then give rise to a four dimensional effective spacetime. Since these are nonsupersymmetric, such compact embeddings are necessarily non-Calabi-Yau. We expect that turning on fluxes would modify the bare masses of closed string tachyons (see e.g. [105]), and more generally the tachyonic physics of the geometry. In the context of codimension three singularities, this could potentially give rise to interesting nonsupersymmetric vacua in four dimensions, with the fluxes potentially stabilizing both tachyons and moduli in the geometry. It would be worth investigating these ideas further.

3.6 Example 6: wrapped branes and non-supersymmetric black holes

The new background that we constructed as (3.58) and (3.65) should more or less capture the full cosmology of our inflationary model, including the slow roll and the exit from inflation. A viable cosmological model, like ours, should also open up avenues to study, for example, black holes and other possible solitonic defects in the model. Black holes can, in principle, form in the early universe (known as *primordial* black holes) during the slow roll or even in the later waterfall stage of inflation [32]. An overproduction of these of course will have disastrous cosmological problems. And therefore their quantity should be negligible. For uncharged black holes this is not much of a problem as most of these black holes eventually all decay via Hawking radiation and therefore they are not there in the present time. However for black holes that are charged, this is not possible as they cannot completely decay. They will radiate till the charge is enough to sustain the mass of these black holes. This way we might be able to see them in the present time.

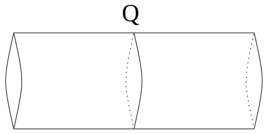
In this example, we would therefore like to ask whether in our supergravity background, it is possible to construct charged blackhole configurations. It turns out that we can exploit the fact that $b_3 \geq 1$ to construct such black holes using D3-branes wrapping three cycles of the non Calabi-Yau deformed conifold.[36] However we have to be careful here. The black holes discussed in [106] are actually BPS and therefore they have the charge-mass equality (i.e the supersymmetry condition). Furthermore, since the masses of these black holes are given by the size of the three cycle, shrinking the size of the three cycle will result in charged massless black holes.[37]

On the other hand, what we have are non-supersymmetric black holes. Therefore the constructions of [106] are not exactly the one relevant here, although our discussion

[36] Not all wrapped branes are black holes in lower dimensions. A generic state of mass M in quantum gravity is a black hole if its Schwarzschild radius is much greater than its Compton wavelength, i.e. $GM \gg \frac{1}{M}$, i.e. for $M \gg M_{pl}$, the state will look like a black hole. Now consider wrapping N Dp-branes on a collection of noncontractible p-cycles with homology basis Σ_i of a compact space that admits p-cycles. This has a charge given in terms of a central charge written schematically as $Z = \sum_i \; q_i \Sigma_i$, where the sum includes dual cycles etc. Consider for simplicity a single cycle. Then the mass of this state in the noncompact 4D spacetime is $M \sim N \cdot T_{Dp} \cdot V_p \sim \frac{NV_p}{g_s l_s^{p+1}}$, where V_p is the volume of the p-cycle and T_{Dp} is the brane tension. If this cycle is supersymmetric, the state is BPS and we have $M \sim |Z|$. For this state to be a black hole in 4D, we require $\frac{N}{g_s} \cdot \frac{V_p}{l_s^{p+1}} \gg \frac{1}{g_s l_s}$ i.e. $N \cdot \frac{V_p}{l_s^p} \gg 1$. Now if $\frac{V_p}{l_s^p} \gg 1$, a p-cycle can be thought of in terms of classical geometry $-$ worldsheet α' corrections are negligible. If we have small $N \sim 1$, this looks like a point particle BPS state since we can potentially violate $M \gg M_{pl}$ if the cycle is not large enough. However if $N \gg 1$, then we necessarily have $M \gg M_{pl}$ even if the cycle volume is not too large, so that the state is indeed a black hole. If the theory has sufficiently low supersymmetry, moduli spaces are lifted typically and one does not need to worry about possible lines of marginal stability where this charge-N state becomes marginally unstable to bifurcation into states of lower charge.

[37] For an analysis of entropy of Calabi-Yau black holes the reader may refer to [107].

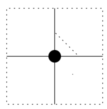
will follow closely the discussion of [106] as the underlying manifold is a warped deformed conifold, but with non-primitive fluxes switched on. In the figure below:



the black hole is the wrapped D3 along the compact direction. The compact direction of the cylinder in the figure above is the three-cycle and the non-compact direction of the cylinder is the four dimensional spacetime. We represent the charge of the wrapped $D3$ as Q. In the $D3/D7$ system, the full cylinder will lie on the world volume of the $D7$ branes, and the $D3$ brane will be along the non-compact direction. Therefore from the world volume of $D3$ brane, the wrapped $D3$ on the compact direction will appear as a black hole of charge Q.

The original configuration of [106] is a supersymmetric one and therefore we have to see whether we can say something about the non-supersymmetric case. The situation is not that bad, because in the absence of fluxes, the background does preserve the required amount of supersymmetry. Therefore, let us first switch off the fluxes. Then we have a static configuration with $D3/D7$ branes on a deformed conifold background, with another Q number of $D3$ branes on the compact three cycle of the deformed conifold. In the earlier sub-section we studied the supergravity solution (albeit with fluxes) for the system, when the wrapped $D3$ branes were *absent*. We now put them in and, in the first approximation, remove the fluxes. Can we write the supergravity solution for this case?

It turns out that when the size of the three cycle is vanishing we can indeed say something about this configuration. This will be the case where we expect to get massless black holes in four dimensions. To study the supergravity solution, let us first T-dualize along directions x and y, where we have already defined (x, θ_1) as one S^2 and (y, θ_2, z) as the three cycle. This T-duality was first studied in [108] and was later detailed in [109]. Our configuration will be more involved than the one presented by [108] or [109], because we have seven-branes in the background. In the limit where $\mu \to 0$ in (3.54), the deformed conifold will turn into a pair of intersecting NS5 branes and the $D3$ brane of the $D3/D7$ system will become a $D5$ brane stretched along x, y directions. Together they form a brane box configuration given below as:



where the box is along x, y directions, and the NS5 branes are denoted as orthogonal lines with $D5$ on the slots of the box (see [110] for a description of brane boxes). The wrapped $D3$ on the three cycle is now a D-string. This is given as the diagonal dotted line on the $x - y$ plane. The black dot in the middle corresponds to the T-dual of the deformed three cycle. This part of this configuration is in fact the original picture of [108]. In our case, however we will have additional branes. This will be the T-dual of the $D7$ branes, which, here will remain as $D7$ branes but oriented along different directions (in the figure above this is rather difficult to depict). So combining everything, we have a brane box like configuration with additional $D7$ branes. Question now is what happens to this configuration if we switch on a B_{NS} field in the original deformed conifold background oriented along:

$$B_{NS} = B_{x\theta_1}\, dx \wedge d\theta_1 + B_{y\theta_2}\, dy \wedge d\theta_2 \tag{3.68}$$

where B_{mn} are defined in (3.67). It is easy to see that this B field will dissolve into metric after two T-dualities along x, y directions, and therefore deform the brane box configuration. This deformation is where we break supersymmetry in this configuration.

In the original case where we had primitive B fields, i.e both the NS and RR three forms selfdual to each other, the T-dualities of the B_{NS} still gave a deformed brane box configuration. But the presence of B_{RR} in fact made the resulting background supersymmetric. Now that we do not have the RR field (or switch on a non-primitive RR field) the configuration becomes non-supersymmetric.

For the supersymmetric case, the spectrum of the string between the NS5 branes was easy to evaluate using standard technique. This gave rise to a *single* hypermultiplet at the intersection region (denoted as the dark circle in the figure above). When the brane box gets deformed, the mode expansion will now no longer be easy to calculate.

On the other hand, the deformation of the brane box itself is not too difficult to calculate. The metric of the intersecting NS5 branes can be written down using standard techniques. Putting D5 branes in the slots of the NS5 branes actually convert this system to an intersecting NS5-D5 brane system. The metric of the box is therefore written using the coordinates dx, dy and $d\theta_i, dr, dz$ (we will not write this here, but the readers can easily work this out). The deformation to the box is therefore the following: in the metric where ever we have dx, dy, we replace this by $dx - B_{x\theta_1} d\theta_1$ and $dy - B_{y\theta_2} d\theta_2$ respectively. An example of this has recently appeared in the last sections of [58], which the reader can look up for a derivation. The presence of $D7$ branes does not modify any of these conclusions (though the metric may get more involved by the back-reactions of the $D7$ branes).

Although the $D7$ branes do not modify the deformation (that we expect in the absence of seven-branes) they do create an obstruction in the system which hinders a simple lift to the type-I scenario. This is where we can compare how our construction differs from earlier studies of related scenarios. The obstruction comes from the fact that we are away from the orientifold point with extra seven-branes that are charged with respect to both the axion as well as the dilaton. A somewhat more clear manifestation of the obstruction is the appearance of extra B fields in the brane box configuration. These B fields have components that are related to the original type-IIB B field (3.68) but they would arise even if we switch off (3.68). In the original framework of [58], where the z coordinate of the deformed conifold was delocalized, these B fields were absent (or can be shown to be pure gauge). However when we have localized metric one cannot gauge these B fields away as they become non-trivial functions of the inherent U(1) fibration of the type IIB metric.

For the configuration of interest, the B fields will appear in the brane box configuration with the following components:

$$B = \alpha_1(dx - B_{x\theta_1}\, d\theta_1) \wedge d\theta_2 + \alpha_2(dy - B_{y\theta_2}\, d\theta_2) \wedge d\theta_1\,, \tag{3.69}$$

where α_i are functions of all coordinates except x and y. From (3.69) we see that even when we switch off $B_{x\theta_1}$ and $B_{y\theta_2}$, there exist non-trivial B fields whose components are determined by α_i. Furthermore from the components we see that the B fields do not lie on any one of the NS5 branes. In fact they have one leg on each of the NS5 and D5 branes.

These aspects make the mode expansion of strings (that are stretched between the NS5 branes) a bit difficult. The physical meaning of the B field (3.69) should now be clear. This is related to the conifold deformation that converts a conifold geometry to a deformed conifold geometry. So starting with a *deformed* brane box configuration (which is the T-dual of a confold geometry with a probe $D3$ and B_{NS} flux (3.68)) we switch on a B field (3.69); and this is the brane box configuration for our case. By construction primitivity is already broken in the original type-IIB framework and therefore this configuration will not preserve supersymmetry. This way we can get the supergravity solution for our case. Of course it still remains a complicated problem to calculate the spectrum of the strings between the two NS5 branes using first principles.

Having gotten the T-dual box configuration, we can use indirect means to estimate the spectrum of the strings inside the box. These strings are of course T-dual of the wrapped D3-branes in the deformed conifold setup. In the absence of fluxes, the wrapped $D3$ brane give rise to massless (or massive, depending on whether μ in (3.54) vanishes or not) hypermultiplets. In the presence of fluxes and also when $\mu \neq 0$, the deformed conifold is T-dual to NS5 brane box with a B field and/or $|\mu|$ separation between the NS5 branes. As we can see, even if the configuration gets deformed by (3.68), the string will always produce massive states. Generic study of B field done earlier in somewhat related scenario showed that these massive states are always hypermultiplets (see [111, 112] for details). These massive hypermultiplets will therefore be the massive black holes that we expect to see in the $D3/D7$ inflationary scenario.

This completes our discussion of the basic picture of black holes from supergravity point of view. Below, we will discuss whether such configurations could be generated in the inflationary D3/D7 setup.

3.7 Example 7: cosmological effects from brane-antibrane annihilation

Now that we have argued a way to get massive charged black holes in our setup by wrapping D3-branes on three cycles, our next question would be whether higher dimensional wrapped branes could also be used to get strings and domain walls in four dimensions. Naively one might think that wrapping D5 branes on either three or four cycles should produce the required defects in our spacetime. On the flip side, formations of domain walls and strings would be in conflict with many cosmological observations. Thus we have to find a way to argue that such defects will not form.

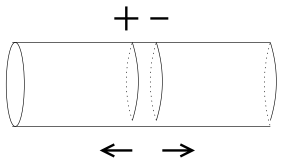
In order to do this, we should go back to the black hole configuration that we studied in the previous example. These black holes are charged objects: therefore to study charged black holes in the inflationary scenario, charge conservation dictates that the only known mechanism is *pair production*. This scenario has been addressed earlier in various papers (see for example [113 − 115]). The black holes in these papers are pair produced as charged point particles.

This simple observation therefore gives us a way to address all the subsequent brane related defects, namely: study them in the framework of pair productions. Therefore we should consider pair productions of $D5/\bar{D}5, D3/\bar{D}3$ and even $D1/\bar{D}1$. The following three conditions would now prevail:

- The pair produced branes can fall back into each other and annihilate. This will not have any observable effects, and therefore can be ignored.
- The pair produced branes are separated by some external forces. The external forces could be via some external fluxes or from inflation itself. The external fluxes that we have are in general not sufficient enough to do this job unless they are very large or have large fluctuation, and therefore the only other mechanism could be inflation itself. As discussed for the point particle cases in [115], the process of inflation can pull these particle-antiparticle pairs so that they do not annihilate any more. This process will keep, say, the particle (i.e the charged black hole) in our cosmological event horizon and hurl away the anti-particle far away so that there will be no causal contact between them any more.
- The pair produced branes do not annihilate completely. This is a new phenomenon that is possible in our set-up because of the special configuration of NS fluxes. There is no counterpart of this effect in the point particle cases and, as we will discuss below, can happen only for some specific branes. Thus it is not universal.

We now begin with the discussion of the second case. In all the cosmological examples we have in mind, the part which is inflating is only along the D3-brane world volume, i.e a de-Sitter metric along $x^{0,1,2,3}$ directions. To an observer along the D3-brane world volume, the brane-antibrane pairs will appear as particle-antiparticle pairs. Therefore the inflationary expansion will be equally responsible here to separate the branes from the anti-branes.

Therefore first consider the creation of $D3/\bar{D}3$ pairs. The pairs wrap the three cycle that is oriented along (y, θ_2, z) and that is orthogonal to the radial direction r and S^2 with coordinates (x, θ_1). In the figure below:



we denote the $D3/\bar{D}3$ pairs by the two cycles with charges $\pm Q$. As before the cylinder is oriented along the three cycle (the compact circle in the figure above) and the space time direction (the non-compact direction of the cylinder in the figure above). The inflationary drag separates the wrapped D3-branes from the $\bar{D}3$ branes, so that we only see, say, the wrapped D3-branes. They appear as charged black holes in four dimensional spacetime.

The above analysis did not consider other orientations for the $D3/\bar{D}3$ pairs. There is no reason why the pairs should only be along the S^3 direction. Since $b_1 = 0$ i.e the first Betti number vanishes, the possibility of getting domain walls from the pairs is not there. It is easy to see that other configurations will not lie on the D7-brane world volume and therefore will not form any wrapped states. Thus the only things that could come out from these pairs are some charged black holes (albeit in very small numbers, which will clear when we apply this mechanism to study primordial black-holes in section 5). The above considerations also rule out any cosmological effects from $D1/\bar{D}1$ pairs, as in the absence of one-cycles ($b_1 = 0$) they will not have any observable consequences.

What happens for the $D5/\bar{D}5$ pairs? It turns out $-$ and this is our third case discussed above $-$ that this situation is a little more involved than the $D3/\bar{D}3$ pairs. To analyze this case let us consider carefully all the possible orientations for the $D5/\bar{D}5$ pairs. First, let the pairs be along directions (x, θ_1) and (y, θ_2, z). The coupling on a D5 brane and an anti D5 brane can be written as [116]

$$\int_{D5} D^+ \wedge (B_{NS} - F_1) - \int_{\bar{D}5} D^+ \wedge (B_{NS} - F_2) \tag{3.70}$$

where F_1 and F_2 are possible gauge fluxes on the D5 and the anti-D5 brane respectively, and D^+ is the four-form of type-IIB theory whose field strength is a five form. The relative sign in the above equation signals the presence of the antibrane. Taking only the component $B_{NS} = B_{x\theta_1}\, dx \wedge d\theta_1$ we see that the above coupling gives us D3-branes with charge Q, where

$$Q = \int_{\mathbb{P}^1} (F_2 - F_1) \tag{3.71}$$

and the integral is over a sphere with coordinates (x, θ_1). Observe that this charge is measured by gauge fluxes induced on the D5 and anti-D5 branes. This is an important difference, because even though both D5 and anti-D5 branes see the same B_{NS} field, they would see different gauge fluxes and therefore would create a D3-brane. This new D3-brane will now wrap the three cycle along (y, θ_2, z) direction and appear as a black hole of charge Q.

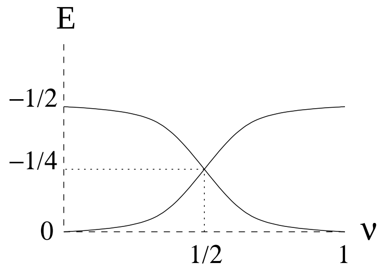
It is now easy to see the two apparent differences from the $D3/\bar{D}3$ pairs. First, the $D3/\bar{D}3$ will annihilate as soon as they are formed, unless the inflationary forces hurl them apart. On the other hand, the $D5/\bar{D}5$ pairs would annihilate to give rise to wrapped D3 of charge Q even before the pair could be separated.

At this point one might wonder about the behavior of the tachyon[38] in this system. Due to the presence of B_{NS} and two different gauge fluxes F_1 and F_2, the mass spectrum is shifted. In fact the zero point energy E, of the string modes between D5 and $\bar{D}5$ is worked out in [112] and can be written as

$$E = \mp\frac{1}{2}\left(|\nu - \frac{1}{2}| \pm \frac{1}{2}\right) \tag{3.72}$$

[38]This is not the D3/D7 tachyon, but the inherent tachyon of a brane-antibrane pair.

where ν is the shift in the mode number because of the fluxes. From this it is easy to study the behavior of the tachyon profile for the system. We can plot E vs. ν and depict the profile as:



It is now clear that there will be *no* tachyon in the system. In fact the tachyon becomes massless and the net result is just D3-branes with charge Q (wrapping the S^3 of the deformed conifold) from the annihilation of the $D5/\bar{D}5$ pairs. All other orientation of the $D5/\bar{D}5$ will not have any observable effects on the D3-brane world volume. Thus we see that other than black holes there is very little possibilities of getting any other charged defects in this scenario. Before ending this section, let us ask what happens for the D3/D7 system in $K3 \times \mathbb{P}^1$ background?

From the analysis we did above, it is clear that we need at least one three cycle to have black holes in four dimensions from wrapped D3-branes. The third Betti number for $K3 \times \mathbb{P}^1$ is zero and therefore will not allow any three cycle. Naively this means that there would be no possibly of getting any charged black holes in four dimensions. What about other wrapped branes? From the Betti numbers of our manifold:

$$\{b_i\} = 1,\ 0,\ 23,\ 0,\ 23,\ 0,\ 1 \tag{3.73}$$

we might expect charged strings and three-branes in four dimensions. These charged strings are extremely rare as they get annihilated by anti strings as soon as they are formed (furthermore, as discussed before, they will have no observable consequences because of the absence of one-cycles). The D3-brane moving towards D7-branes will only detect them at the final stage of the inflationary scenario. On the other hand, due to compactness of the internal manifolds, these charged three-branes are not allowed to form in our case. Also since $b_1 = 0$ there would be no domain walls (to get domain walls we need D3 branes to wrap one-cycles and/or D5 branes to wrap three cycles and/or D7 branes to warp five cycles. All of which are absent here). Therefore it seems that D3/D7 inflationary model in $K3 \times \mathbb{P}^1$ background has zero possibility of black holes formation from wrapped branes. However before we come to a definite conclusion let us contemplate another mechanism that could in principle allow some possibilities of black holes formation. This mechanism has to do with the choice of three form fluxes in this background.

The fluxes on the K3 manifold − on which we also have wrapped D7-branes − induce a configuration of intersecting D5 branes on the D7-branes world volume. These D5 branes intersect along the $3+1$ space which forms our spacetime. This world volume configuration is in fact U-dual to the deformed conifold setup that we studied above! The wrapped D3-branes in the earlier configuration will be U-dual to fundamental strings between the two

D5 branes. However there is one obvious problem with this map: we cannot separate the two D5 branes, and therefore the strings between them will only contribute massless hypermultiplets in four dimensions. One way out of this is to allow different gauge fluxes on the two D5 branes. This will shift the zero point energies of these strings.

In practice however it turns out more difficult to justify these massive strings in the intersecting brane configurations. Therefore the D5 brane configurations also will not be very useful to allow the formations of black holes in the $K3 \times \mathbb{P}^1$ background.

Before ending this section, let us contemplate another phenomenon as the D3-D7 open string tachyon condenses: This is the possible formation of non-BPS branes wrapped on appropriate cycles of the compact geometry. We recall [117] that the spectrum of D-branes in type-IIA (B) theory included odd (even) dimensional uncharged D-branes, besides the more familiar BPS D-branes that carry RR-charges. These non-BPS branes have an open string tachyon on their worldvolume and are unstable. While their tensions also scale as $\mathcal{O}(\frac{1}{g_s})$, they are dressed by the open string tachyon potential which appears in the corresponding DBI action. Thus for instance the IIB theory on $K3 \times \mathbb{P}^1$ in principle will nucleate non-BPS 2-branes wrapped on 2-cycles therein, since they are uncharged objects allowed simply on energetic grounds as the D3-D7 tachyon condenses (on the other hand, the approximately deformed conifold does not admit 2-cycles so this process is not possible here). As possible defects, these unstable branes would naively be expected to decay rapidly due to the dominant open string tachyon. From the point of view of string theory, the work of [118] (using Sen's boundary state [119]) shows that the dominant decay mode for these unstable branes is into the full tower of massive closed string states. On the other hand, it is *a priori* unclear what the effective description of such decay processes is, via supergravity. While one could naively imagine these wrapped unstable branes to appear as point-like uncharged black holes in the noncompact four dimensions, such objects would typically also source the dilaton and other scalar fields. Thus it is not evident that these systems describe black holes with smooth horizons as opposed to naked singularities. Related work on possible supergravity descriptions of brane-antibrane stacks appears in e.g. [120]. It would be interesting to understand these questions in greater detail.

4. Classification and stability of cosmological solutions

In section 3 we studied various cosmological examples, by taking different limits of the background fluxes and with some quantum corrections. We also saw the necessity of higher derivative corrections to get any reasonable answers. However a precise analysis of these higher derivative terms has not been worked out in the literature, and therefore the solutions that we presented in sec 3.1 to sec 3.3 are only approximate. Furthermore these solutions are only valid in the regimes discussed for various individual cases respectively.

In the absence of an exact solution, we would then like to classify the possible background metrics that we can get using two and three warp factors. The metric that we get in type-IIB is of the following generic form:

$$ds^2 = \frac{f_1}{t^\alpha}(-dt^2 + dx_1^2 + dx_2^2) + \frac{f_2}{t^\beta}\, dx_3^2 + \frac{f_3}{t^\gamma}\, g_{mn}\, dy^m dy^n \tag{4.1}$$

where $f_i = f_i(y)$ are some functions of the fourfold coordinates and α, β and γ could be positive or negative number. For arbitrary $f_i(y)$ and arbitrary powers of t, the type-IIB metric can in general come from an M-theory metric of the form (2.33) with three different warp factors A, B and C, given by:

$$
\begin{aligned}
A &= \frac{1}{2}\log\frac{f_1 f_2^{\frac{1}{3}}}{t^{\alpha+\frac{\beta}{3}}} + \frac{1}{3}\log\frac{\tau_2}{|\tau|^2}\,, \\
B &= \frac{1}{2}\log\frac{f_3 f_2^{\frac{1}{3}}}{t^{\gamma+\frac{\beta}{3}}} + \frac{1}{3}\log\frac{\tau_2}{|\tau|^2} \\
C &= -\frac{1}{3}\left[\log\frac{f_2}{t^{\beta}} + \log\frac{\tau_2^2}{|\tau|}\right]
\end{aligned}
\tag{4.2}
$$

where we have isolated the complex structure dependences, and $\tau = \tau_1 + i\tau_2$ as before. The above is therefore what we talked about in the beginning of section 3, but clearly there are many cases with only two warp factors. The two warp factor cases would be when $B = C$, and A different. To see what the possible choices are for such a background, we need to find the difference $B - C$. This is given by:

$$
B - C = \frac{1}{2}\log\frac{f_2 f_3}{t^{\gamma+\beta}} + \log\frac{\tau_2}{|\tau|}\,, \tag{4.3}
$$

which could be simplified further if we consider a background with trivial axion-dilaton i.e $\tau = i$ with the seven-branes charges cancelled locally.[39] From here, the two warp factor case would be when (4.3) vanishes. Since the space and time dependent parts of (4.3) can be isolated, (4.3) can only vanish if

$$
f_2 = f_3^{-1}\cdot\frac{|\tau|}{\tau_2}\,, \qquad \gamma + \beta = 0 \tag{4.4}
$$

with α and $f_1(y)$ remaining completely arbitrary. To simplify things a little bit, let us also consider $\tau = i$ which means that we are keeping the seven branes far away and also taking the limit when the dilaton is zero (see section 3 for the precise equations for which such a situation could be realised). Then the condition (4.4) would fall into the following cases:

Case 1: $\gamma = \beta = \alpha = 0,\ f_2 = f_3^{-1}$

This is completely independent of time, and therefore the warp factors are functions of the fourfold coordinates. We now expect that the warp factors A and B would be related as $A = -2B$, as we have seen in section 2. Using the general forms of A and B in (4.2), $A = -2B$ would immediately imply $f_1 = f_2$. This is therefore the supersymmetric case studied earlier in various papers including [64, 121, 122]. The M-theory lift of this will have the following warp factors:

$$
A = \frac{2}{3}\log f_1\,, \qquad B = C = -\frac{1}{3}\log f_1\,, \tag{4.5}
$$

which is of course the fourfold solution of [79].

[39] This is when the M-theory fiber is a $\mathbb{Z}_2$ orbifold of T^2.

Case 2: $\gamma = \beta = 0,\ \alpha \neq 0,\ f_2 = f_3^{-1}$

There could be various choices here when $\alpha \neq 0$. All of them are time dependent cases and therefore break supersymmetry. The case where $\gamma = \beta = 0, \alpha = \pm 2, f_1 = f_2 = f_3^{-1}$ is our Examples 1 and 2 earlier (in section 3.1 and section 3.2). As discussed there for the case when $\alpha = 2$, this is an approximate background when we take the background G_{mnpq} fluxes to be very small on an almost non-compact fourfold. For the case when $\alpha = -2$, we were not required to take fluxes very small but solutions there only existed *iff* higher derivative corrections are put in. In both cases the membrane is moving very slowly and the warp factors in M-theory are (henceforth we will keep $t_o = 0$ for simplicity)

$$A = \frac{2}{3}\log f_1 \pm \log t\,, \qquad B = C = -\frac{1}{3}\log f_1\,. \tag{4.6}$$

The next corresponding case would be when $\alpha \neq \pm 2$ with f_1 may or may not be equal to f_2. With the analysis done in section 3, this condition seem not to be allowed to the order that we took the quantum corrections. It could be that these cases are realized when we certain terms in the background equations of motion are ignored. It will be interesting to study these possibilities.

Case 3: $\gamma = -\beta, \qquad f_2 = f_3^{-1}$

Again there are a few choices here. The case studied in Example 3 earlier (section 3.3) is when $\alpha = \beta = -\gamma = 1,\ f_1 = f_2 = f_3^{-1}$. For this case unfortunately the internal G-fluxes G_{mnpq} become time dependent. Therefore this case seems to be a viable solution only when we remove the branes from our picture. Thus this case is basically outside the D3/D7 inflationary setup, unless we incorporate corrections to the anomaly equation (2.55) which would allow such kind of fluxes. One interesting case to consider would be to find a de-Sitter like solution here. From our earlier analysis (2.20), we know that type-IIB de-Sitter can be gotten from dimensional reduction over a six dimensional space, if the warp factors satisfy:

$$\beta + 3\gamma = 2 \qquad \Longrightarrow \qquad \alpha = \beta = -1\,, \qquad \gamma = 1 \tag{4.7}$$

which is related to what we got in (3.42) by the replacement $t \to \frac{1}{t}$ in (3.42). Therefore for the two cases we can consider fourfolds in M-theory with warp factors given by:

$$A = \frac{2}{3}\left[\log f_1 \pm \log t\right], \qquad B = C = -\frac{1}{3}\left[\log f_1 \pm \log t\right] \tag{4.8}$$

with the $+$ sign giving a de-Sitter solution and the $-$ sign giving the cosmology studied in section 3.3. In the latter case, the fourfold is effectively non-compact. Similarly there are other interesting cases which could possibly be seen under some restricted choices of flux backgrounds.

These are therefore all the possible allowed choices with two warp factors. We now would like to come to the case with three warp factors. For generic metric the warp factor choices are already given in (4.2). Of course the three warp factor cases are much more flexible, but on the other hand the background equations of motion are pretty involved now. We will perform a more systematic study, along with possible derivations, of this in a sequel to this paper.[40] For the time being let us analyse all the possible choices within this scenario.

Case 4: $\alpha = \beta = 2$, $\gamma = 0$ $f_1 = f_2$

This is by far the most interesting case. The internal six manifold is time independent. From our earlier analysis (2.20), this example would correspond to an exact de-Sitter background. And therefore this would be an accelerating universe with the three warp factors given by:

$$A = \frac{2}{3}\log\frac{f_1}{t^2}, \qquad B = \frac{1}{2}\left[\log f_3 + \frac{1}{3}\log\frac{f_1}{t^2}\right], \qquad C = -\frac{1}{3}\log\frac{f_1}{t^2}. \tag{4.9}$$

We see that the internal fourfold has time dependent warp factors although the type-IIB six dimensional space is completely time independent. Such a background has the advantage that the four dimensional dynamics that would depend on the internal space will now become time independent.

Case 5: $\alpha = \beta$, $\gamma \neq 0$ $f_1 = f_2$

Although interesting, case 4 is of course not the most generic way to get a 3+1 dimensional de Sitter space. If we remove the restriction that the internal space should be time independent, then there are many other choices. From (2.20) these cases can be classified as:

$$\alpha = \beta = \kappa, \qquad \gamma = \frac{2-\kappa}{3}, \qquad f_1 = f_2, \tag{4.10}$$

where κ could be any number. We have also put no restriction on f_3, although one might want to keep $f_3 = f_2^{-1}$ to comply with the supersymmetric case. All these cases would give rise to accelerating universes of course, with warp factors

$$A = \frac{2}{3}\left[\log f_1 - \kappa\log t\right], \qquad B = \frac{1}{6}\left[\log(f_1 f_3^3) - 2\log t\right], \qquad C = -\frac{1}{3}\left[\log f_1 - \kappa \log t\right]. \tag{4.11}$$

Other cases with γ not equal to the one above (4.10), therefore will not be de-Sitter, but could be either accelerating or decelerating universes. Furthermore, many of the earlier cases we studied with two warp factors may also be studied with three warp factors if we do not put any restrictions of f_1, f_2 and f_3. This completes the classification.

Let us now briefly turn to the issue of moduli stabilization. This is a bit subtle when the internal manifold becomes time dependent, as sometimes the internal fourfold could have time-dependent warp factors but the internal threefold in type-IIB has time independent warp factors. The various cases can be classified as follows:

- The internal fourfold in M-theory and internal threefold in type IIB both have time-independent warp factors.

[40] Of course we should also consider the cases with four (or maybe more) warp factors too. Having four warp factors would mean that the type IIB six manifold is by itself a fibered product − with the fiber (of some dimension) warped differently than the base. Since not all manifolds can have this inherent property, it will be interesting to analyse this case in details to see what possible cosmology comes out from here.

- The internal fourfold has time-dependent warp factors whereas internal threefold in type-IIB has time-independent warp factors.

- Both the internal fourfold in M-theory and the internal threefold in type-IIB have time-dependent warp factors.

The first case of course incorporates the supersymmetric background where, unfortunately, there are still no models with all moduli stabilized. Therefore the non-supersymmetric cases will definitely be much more complicated to handle. This problem becomes even more severe if the internal G-fluxes G_{mnpq} in M-theory are time dependent. Fortunately, some of the toy cosmologies that we studied in sec 3.3 have G-fluxes that are time independent. These solutions are not de-Sitter, but in case 4 above we gave an example where, with three warp factors, it might be possible to realize a de-Sitter background. Although a full analysis of this background is beyond the scope of this paper, we will assume that such a background may also allow a time independent G_{mnpq} flux on the fourfold.

Now the situation is much more tractable. The G_{mnpq} fluxes will induce a superpotential [63] that will fix all the complex structure moduli of the fourfold. Other Kähler structure moduli may be fixed by non-perturbative effects. Some discussion of this has appeared in [123, 72]. From type-IIB point of view, these non-perturbative effects are related to the gaugino-condensate on the D7-branes[41] or euclidean D3-brane instantons. The original KKLT construction [35] of de-Sitter vacua uses this mechanism to fix the Kähler moduli, and an anti-D3-brane to break supersymmetry. For our case we broke supersymmetry *spontaneously* using non-primitive fluxes. The second case could probably be dealt with from the type-IIB point of view, where the six manifold has a time independent warp factor. However it is not clear whether the fluxes could always be made time independent. This then leads us to the question of moduli stabilization when the fluxes themselves are varying wrt time. One possible explanation could be that at a given time $t = t_0$, one fixes the fourfold moduli using a superpotential. Then, as the fluxes vary with respect to time, the moduli are allowed to vary accordingly. Again, no analysis has yet been performed to this effect that a concrete statement could be made. We will therefore leave this discussion here and continue it in the sequel.

The third case is intractable so far, as the time dependences of the internal manifolds in M-theory as well as in type-IIB theory make it hard to give a physical picture regarding modulus stabilization. As mentioned in the Introduction, the transient accelerations à la [6] are too short to be used for slow-roll inflation.

[41] It is not too difficult to identify the gaugino condensate terms from M-theory. In M-theory the singularities of the fiber are related to the positions of the seven-branes (not all are D7 though). On the other hand, G-fluxes can be decomposed into a *localized* and a *de-localized* part. The de-localised part eventually becomes the H_{NS} and H_{RR} in type-IIB theory [64, 71], whereas the localized part becomes the seven-brane gauge fields [64, 71]. In M-theory, the localized G-fluxes couple with the supergravity fermions. Expectation values of these fermion terms give rise to gaugino condensates on type-IIB D7-branes creating a superpotential. More details on this will be presented elsewhere.

5. Primordial black holes in cosmological backgrounds

We now come to another interesting cosmological phenomenon: the formation of primordial black holes in the early universe. As studied in the literature, there could be various possibilities. Black holes may form from the large density perturbations [124, 125] induced near the end of the inflationary era. This is basically one of the key mechanisms of black hole formation. We will soon show how the the $D3/D7$ model may realize some aspects of this. In a parallel scenario the formation of black holes starts when the mass of the black hole is equal to the mass of the horizon.

But there are other mechanisms. In some inflationary models that are proposed in the literature, inflation ends by bubble nucleation. This is a phase transition − a strong first order phase transition that proceeds explosively by the bubble nucleation. Due to this phase transition primordial black holes can easily form. Details of this mechanism have appeared in [126].

Another mechanism is related to the thermal fluctuation during the very early universe. The productivity rate of the black holes is particularly efficient at very high temperature. A discussion of this has appeared in [127].

These black holes formed in the early universe (via any one of the mechanisms listed above) are usually light enough so that Hawking evaporation can take place. The lifetime T of such a black hole of mass m is given by the celebrated formula of Hawking [128]

$$T = \frac{m^3}{M_p^4} \cdot \frac{1}{N} \tag{5.1}$$

where $M_p = 1.22 \times 10^{19}$ GeV is the Planck mass, and N denotes the number of particle degrees of freedom into which the black hole can decay. Using the above relation, a simple calculation tells us that a black hole of mass $m \approx 10^{12}$ kg will have a lifetime equal to the age of our present universe (i.e 10^{10} years). These black holes therefore will now be evaporating. Lighter black holes would have evaporated long ago. What about heavier black holes? These black holes would still be present and therefore would contribute to the energy density of the universe. Recall that if black holes form early enough, they can easily dominate the energy density of the universe irrespective of whether they inhabited a very tiny fraction or not at the beginning. Thus those heavier black holes should not have energy density more than the critical density of the universe. This is an important constraint. An interesting possibility here is that, if the energy density of these black holes is greater than the critical density, they might be a candidate for cold dark matter. The black holes that are evaporating today are also constrained by the critical energy density. In fact studies of the γ ray backgrounds [124, 129] limits black holes in this mass range to contribute at most some orders of magnitude below the critical density. The constraint can get a little weakened if one assumes the formation of relics [130].[42]

[42] As a parallel comment, note that classical black holes induced by metric perturbations, which were in turn generated during the water-fall stage of the hybrid inflation, can be sufficiently abundant − since the inflation is nearing its final stage − to be dark matter candidate. An analysis of this was done in [131]

From the discussion above one might wonder if there could be any constraints on those black holes that have evaporated away completely, i.e those black holes that have masses much below 10^{12} kg. It turns out that these black holes are strongly constrained [132, 133]. The constraint comes from the fact that for black holes of masses $m \approx 10^7$ kg would be evaporating during nucleosynthesis. Since nucleosynthesis is an important part of the formation of the present universe, an overproduction of these light black holes would interfere with the process. Therefore these black holes are also strongly constrained. For further details the readers may want to see the analysis given in table I and II of [132].

Before we end this section we should discuss the recent extension to the standard cosmological scenario: the so called thermal inflation [134]. This is a second period of inflation which is triggered by supersymmetric fields that have an almost flat potential. In fact this second period has nothing to do with the usual inflationary regime that solves the horizon, flatness and the monopole problems. However, similar to the usual inflation, the scalar fields that drive the thermal inflation also have a false vacuum energy. Once the temperature drops to a certain value, the false vacuum energy drives the new inflation till the temperature drops further down and takes the scalar field out of the false vacuum.

Identifying the scalar fields (that drive the new thermal inflation) as the moduli fields of string theory (i.e the scalars that would come from the non-trivial Betti numbers of the internal manifold) would in fact lead to the famous moduli problem (also known as the Polonyi problem) [135]: namely, their expectation values of order M_p would create many particles that remain till the nucleosynthesis and therefore would be inconsistent with cosmological predictions. Many solutions to this problem have been proposed [136]. All of them are more or less based on the fact that the thermal inflation, which is basically a period of late inflation with Hubble expansion parameter of the order of the weak scale, can effectively solve this problem (see the second reference of [136] for more details). The dilution effect of the thermal inflation should also be small enough so that it doesn't effect the density perturbations generated by the original inflationary period.

The question now is to see the effect of the second stage of inflation on the primordial black holes. It turns out that the main effect of thermal inflation is to dilute the density of the primordial black holes. Furthermore, during the two periods of inflation (i.e the original one at the beginning and the later, thermal, one) a given primordial black hole production corresponds to a later stage in the original density perturbation generating epoch of inflation [132]. In addition to that the thermal inflation introduces missing mass ranges for the black holes by allowing those co-moving scales to be pulled outside by thermal inflation that had originally entered *before* thermal inflation had set in. Also

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following a *generalized uncertainty principle* that can be inspired from string theory. It was shown therein that these primordial black holes (which are still extremely small, like 10^{10} times the Planck mass) can stop their Hawking evaporation when they reach the Planck size. These will then contribute to the dark matter. Based on hybrid inflation, such primordial black holes remnants can indeed saturate the dark matter Ω_m. For more details the readers may want to see [131].

since the energy scale of the thermal inflation is much lower than the original inflationary period, any new density perturbations are expected to be small. This would mean that new black holes are not formed, after re-entering the horizon again (see [124, 132] for more details).

Having summarized some of the details related to the formation of black holes in the early universe, it is now time to continue our discussion of these issues in the hybrid inflationary model. This will help us connect the picture with the wrapped $D3$ brane intuition that we described earlier.

5.1 Primordial black holes in D3/D7 system?

To use the ideas we developed earlier to study black holes in our set-up, we have to give some reasons for the formation of brane-antibrane pairs in this scenario. A set of arguments that motivates to use the machinery that we developed in sec 3 is as follows:

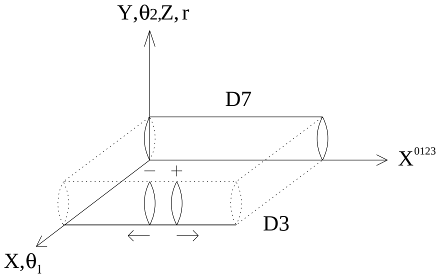
- As the D3-brane is moving towards the D7-branes, there is a perturbation in the background due to the time dependent evolution of background fields (recall that we have both H_{NS} as well as H_{RR} fields that are evolving with time). If these fields are fluctuating fast enough, then the time dependent perturbations will be responsible in creating brane-antibrane pairs.

- As the D3-brane approaches the D7-branes, the open strings stretched between them become tachyonic. These tachyonic instabilities will signal a phase transition. Condensation of these tachyons pumps energy into the system resulting in copious production of brane-antibranes. From the dynamics of the system, one would expect this to dominate over the first one.

From either of the two cases, once we assume the existence of brane-antibrane pairs, the rest of the discussion should follow as given earlier. In fact we studied this kind of background in sec 3 and sec 4 (as case 4) above, where $\alpha = \beta = 2, \gamma = 0$ and $f_1 = f_2$. Now if we replace g_{mn} in (4.1) by the metric (3.65), then this will be a precise background that allows us to study primordial black holes in the inflationary scenario. Thus combining, (3.65), (4.1) and (4.9), our background will take the following simple form:

$$\begin{aligned} ds^2 = \frac{f_1}{t^2}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + f_3 \Big[& h_1(dz + a_1 dx + a_2 dy)^2 + h_2(dy^2 + d\theta_2^2) + \\ & + h_6\, dr^2 + h_4(dx^2 + h_3\, d\theta_1^2) + \\ & + h_5 \sin\psi(dx\, d\theta_2 + dy\, d\theta_1) + \\ & + h_5 \cos\psi(d\theta_1 d\theta_2 - dx\, dy) \Big] \end{aligned} \tag{5.2}$$

where h_i are given in (3.66) and the background threeform by (3.67). As we see both the internal space and the background fluxes are time independent. The fluxes are non-primitive and they break supersymmetry. The above metric therefore captures the complete scenario wherein primordial black holes are created in the D3/D7 inflationary model. This is expressed by the following figure:



In this figure, the compact space is along $(x, \theta_1, y, \theta_2, z, r)$, with three-cycles denoted by coordinates (y, θ_2, z). The seven-branes therefore wrap the directions (y, θ_2, z, r) and are stretched along de-Sitter spacetime $x^{0,1,2,3}$. We denote the surface of the D7-branes by a cylinder, whose compact directions are along (y, θ_2, z) and also r. The world volume of D3 is along $x^{0,1,2,3}$. Both the D7-branes and the D3-brane are points on (x, θ_1) space, and the D3-brane is slowly moving towards D7-branes with a velocity $\dot{y}^m$ (that we determined earlier for various cases). As the D3 moves, there would be creation of small number of $D3/\bar{D}3$ pairs. These pairs are quickly separated by the inflationary expansion, as depicted in the figure above. Finally, when the D3-branes comes very close to D7-branes, condensation of D3-D7 tachyons pumps energy into the system creating more $D3/\bar{D}3$ pairs.

Of course, this is not the only mechanism working here. Instead of the inflationary expansion separating D3-branes from antibranes, we could have D3-branes created directly from D5 brane-antibrane pairs by the background B_{NS} fields. The production rates for this are expected to be bigger than for the D3 pair productions, as we discussed before.[43] From either of these two mechanisms, black holes in 3+1 dimensional de-Sitter space will appear as D3-branes wrapping a three cycle (y, θ_2, z) with the intrinsic metric

$$g = \begin{pmatrix} h_1 + h_2 & 0 & a_2 h_1 \\ 0 & h_2 & 0 \\ h_1 a_2 & 0 & h_1 \end{pmatrix} \tag{5.3}$$

and with masses determined by the volume of the wrapped cycle. One can now estimate the productivity rates of black hole formation using two different techniques: (1) Evaluate the productivity rates from an effective 3+1 dimensional theory by dimensional reduction over the threefold that we discussed above, or (2) Evaluate the productivity rates of brane-antibrane in a fluctuating background directly from ten dimensional type-IIB theory. Both of these techniques should yield the same answer. However here we will briefly sketch the first mechanism which is by now a standard way to derive the productivity rates of charged black holes in any dimensions. For more details the reader is referred to [115, 114].

[43] The $D5/\bar{D}5$ pairs do see some B_{NS}-fluxes, so that pair production for them is not tied to tachyonic modes near the D7. And there will be some probability for the D3 defect to be produced when the D5s annihilate − note that the dominant decay mode (i.e the case when $F_2 \approx F_1$) for the D5s is still into just radiation with no lower dimensional brane remnants. Thus this rate will be smaller although nonzero. Therefore the production rates of black holes from the $D5/\bar{D}5$ annihilations are expected to be higher than the ones from separating the $D3/\bar{D}3$ pairs by the inflationary expansion.

The analysis of [115, 114] starts by identifying *two* different instanton actions $I_{\rm dS}$ and $I_{\rm BH}$ that are responsible for the production rates:

- $I_{\rm dS}$ is the euclidean action of four-dimensional gravitational instanton that mediates the nucleation of a de-Sitter space from nothing. In terms of our language, this would be the creation of the four-dimensional part of the D3/D7 metric (5.2) via an instanton from nothing. This four-dimensional part is of course an effective theory when we reduce type-IIB over the non-Calabi-Yau background.

- $I_{\rm BH}$ is the euclidean action of the gravitational instanton that mediates the pair creation of charged black holes in four-dimensions from nothing. Again, in terms of the analysis that we did earlier, this would now be the dimensional reduction of the wrapped $Dp/\bar{D}p$ case with $p = 3, 5$ in the D3/D7 setup.

Thus, in the language of [115], we have two different instantons: one for the background and the other for the objects in this background. The productivity rate Γ is now given by

$$\Gamma = \eta \, e^{-2(I_{\rm BH} - I_{\rm dS})} \tag{5.4}$$

where η is the one-loop contribution from the quantum quadratic fluctuations in the background fields [137, 127]. A precise value of η is not very relevant for us. What we require are the values of the instanton actions.

The instanton actions have been calculated in many papers (see for example [115] and citations therein) and for black holes of different charges. Our effective theory in four dimension will be the same except that we now expect many scalars from the moduli of the theory. The potential for these scalars will come from either [63] (in M-theory) or [138, 121] (in type-IIB theory) or [71, 66] (in heterotic theory), including non-perturbative corrections. Assuming that the potentials fix these scalars to some expectation values, we can integrate them out to get our instanton actions. The final productivity rates from all these analyses turn out to have a universal form, and is given by:

$$\Gamma \sim e^{-\frac{m}{\sqrt{\Lambda}}}, \qquad m = \frac{T}{g_s} \int dz dy d\theta_2 \sqrt{h_1 h_2 (h_1 + h_2 - a_2^2 h_1)} \, f_3^3 \tag{5.5}$$

where m is the mass of the black holes coming from the wrapped D3-branes with tension T, h_i are given in (3.66) and g_s is the type-IIB coupling constant (i.e the value of the dilaton that comes from the non-trivial complex structure τ of the fiber torus (2.46) in M-theory). Finally, Λ is the cosmological constant that would appear from the inherent non-commutativity on the D7-branes [25] (see also [139] for a more updated discussion on this).

Although a full numerical analysis of productivity rates will now require the values of the warp factors f_1 and f_3 − which can only be determined once we know the metric using three warp factors − a qualitative analysis (by evaluating (5.5) with known values of h_i (3.66) and f_1, f_3 satisfying $f_1^{-1} = f_3 \approx \sqrt{j}$ as in (3.46)) easily tells us that Γ is very small here. This is of course consistent with the inflationary scenario.

But this is not the full story yet. As the D3-brane moves towards D7-branes the productivity rates − which are initially given by given by Γ above − will get further enhanced. This is because, when the D3 finally reaches D7 the open string tachyon between D3 and D7 condenses pumping energy to the system, thereby enhancing the productivity rates of brane-antibranes. This will continue till the moving D3-brane finally dissolves into the D7-branes as a non-commutative instanton. We will not perform a detailed analysis of Γ here, but a qualitative analysis will tell us that the productivity rates are still very small, because the last stage of the inflationary process where the tachyon condenses and the D3 is finally absorbed in the D7 branes is very short.[44] On the other hand, we saw earlier that the D3/D7 system on $K3 \times \mathbb{P}^1$ has zero productivity rates. Therefore with a non Calabi-Yau threefold as an internal space and $b_3 \geq 1$, there is a finite (albeit very small) probability of detecting charged black holes in four-dimensional spacetime.

6. Semilocal defects with higher global symmetries in $D3/D7$ model

Having studied various cosmological solutions in the D3/D7 system, let us now turn our attention to something a little different from our earlier constructions, namely the semilocal defects that can arise in some regimes of the D3/D7 model. In our earlier paper [30] we addressed the issue of semilocal strings. Semilocal strings were discovered by Achùcarro and Vachaspati in the early nineties [51]. These strings defy all the standard lore of the stability and existence of solitons. In fact they are *non-topological* solitons and their existence can occur in theories that have both global and local symmetries.

The standard analysis of the classification of solitons is based on the topology of the moduli space and the corresponding homotopy groups. In the presence of global symmetries (shared by, say, only the moduli), the above naive classification can fail. This is where certain non-topological objects, so called semi-local defects, can surface. These defects could be made stable by restricting the Higgs masses to lie between some range of energies. For one dimensional defects − the semi-local strings − it was shown by Hindmarsh [52] that when the Higgs mass is smaller than that of the gauge bosons, the semi-local strings are absolutely stable despite the fact that the naive vacuum manifold has a trivial first homotopy group. In fact for the theory having a symmetry group G at *vanishing* gauge coupling, the homotopy classification for semi-local defects can sometime be done if we consider a much smaller subgroup G_s that is in principle gauged when we switch on a non-trivial coupling (or alternatively switch on the gauge fields). The classification then takes a somewhat familiar form:

$$\pi_n\left(\frac{G_s}{H_s}\right) = \mathbb{Z}\,, \qquad \pi_n\left(\frac{G}{H}\right) = 1 \tag{6.1}$$

for a given n where n determines the kind of defects one would see in a given model and H_s is the unbroken subgroup of G_s. Using this one can show that, although the vacuum

[44] In fact the interval δt can be estimated for our case once we know the D3 brane velocity $\dot{y}$ at the last stage when it is near the D7 branes, as $\delta t \approx \frac{d_c}{\dot{y}}$. Here d_c is the critical distance at which an open string tachyon is formed and is given by [25] $d_c \equiv \frac{1}{2}\pi\alpha' B_{y\theta_2}$ where $B_{y\theta_2}$ is the B_{NS} field from (3.67) with other components vanishing. Since we expect $\dot{y}$ to be large (its not slow roll anymore) and d_c of order string scale (from above); the absorption process is almost instantaneous resulting in a very small productivity rate of the blackholes from this process also.

manifold may have trivial homotopy, the theory could still have stable defects. A more detailed analysis has been carried out in [52, 140] to which the reader is referred for further details.

It turns out that the D3/D7 model that we studied here is a good place to realize these defects as non-topological solitons of the theory,[45] although the regime we shall use to study these solitons may not necessarily be the inflationary one. This is because of the existence of global symmetries, which generically require many scalars that would typically ruin the inflationary nature of the model. Furthermore there are few more points that we have to take care of before we can embed non-topological defects in our theory:

- The dynamics of the seven branes should be isolated from the dynamics of the D3 branes. As discussed in [30] for the case when the internal six manifold is of the form $K3 \times \mathbb{P}^1$, the seven branes wrap the K3 manifold which can be fixed to a large size. For our case the seven manifold wrap a four-cycle of the metric (3.65) oriented along (z, y, θ_2, r), whose unwarped metric can be written as

$$g_{mn} = \begin{pmatrix} h_1 & a_2 h_1 & 0 & 0 \\ a_2 h_1 & h_2 + a_2^2 h_1 & 0 & 0 \\ 0 & 0 & h_2 & 0 \\ 0 & 0 & 0 & h_6 \end{pmatrix} \tag{6.2}$$

  where h_i and a_2 are defined in (3.66). Thus starting with the seven brane coupling g_7^2, the effective three brane coupling can be easily deduced to be

$$\frac{1}{g_3^2} \equiv \frac{1}{g_7^2} \int_{\Sigma_4} e^{4B} h_2 \sqrt{h_1 h_6} \tag{6.3}$$

  where Σ_4 would be the four-cycle with the metric g_{mn} (6.2) on which the seven-branes are wrapped. Using arguments of moduli stabilisation, (as we would require to fix the size of the internal manifold against any runaway) this effective coupling can be made very small and therefore the seven branes would effective decouple from the three brane dynamics.

- The fourfold we use to describe our new six-manifold should have a representation in terms of a Weierstrass equation. In fact, this is exactly what we achieved when we derived our background in example 5 of sec 3. As mentioned therein, the Weierstrass equation is written as $y^2 = x^3 + xf + g$ with $f \oplus g$ being sections of $\mathcal{L}^4 \oplus \mathcal{L}^6$ with $\mathcal{L}$ being $\mathcal{K}_{\mathcal{B}_\mu}^{-1}$, and $\mathcal{B}_\mu$ as in (3.54). Thus the fiber would have to degenerate over a subspace of our six manifold (it could be the full six manifold, but then the interpretation of the singularities in terms of type IIB branes becomes complicated).

The next question therefore would be to specify this subspace, along with its possible metric. This is not difficult. We want the seven branes to be points in the (x, θ_1) directions.

[45]Semilocal vortices based on unitary symmetry groups have been constructed in [141] using Hanany-Witten type brane configurations [142] in type-IIA string theory.

The metric along these directions can be worked out from (3.65). The result is a warped torus with the following metric:

$$ds^2 = \frac{e^{2B}}{C}\,|dz|^2 = \frac{e^{2B}}{C}\left[dx^2 + (C - \beta_1^2 E^2) C\, d\theta_1^2\right], \tag{6.4}$$

where all the quantities appearing above are defined in (3.66), and the complex structure of the torus can be easily inferred to be $\tau = i\sqrt{(C - \beta_1^2 E^2)C}$.

From the above metric (6.4) it is now possible to see how the seven branes are placed. Observe that the warp factor e^{2B} will contain the information of the seven-branes, as the other warp factors are known in terms of the variables (E and F) before the geometric transition. From the M-theory view point, the warp factors satisfy an equation of the form (2.55) with the condition (2.47) (for the two warp factor case). Of course the kind of metric that we want can only be derived from three warp factors by choosing case 4 of sec 4, and therefore the warp factors would change suitably. On the other hand we could also study this from type-IIB point of view. An analysis of warp factors (for the supersymmetric case) has been done in [122]. The result is presented in terms of Green's functions that pick up singularities at the points where we expect seven branes and three branes. We will not go into the details of those calculations because our system is a little more complicated due to the non-primitivity of the background fluxes, instead we will try to estimate the possible values of e^{2B} by going to a regime where we could apply the exact result of [143].

The regime that we are interested in will be when the complex structure of the base is $\tau \approx i$, and the fluxes are small there (so that the warp factor equation is solely affected by the brane singularities). In terms of the variables used to define the metric (6.4), this is possible if we restrict ourselves to the region:

$$\frac{1}{\alpha_0}\left(C - \frac{1}{C}\right)\left(D - \frac{1}{D}\right) + C^2 - \alpha_0 D E^2 e^{2\phi} = 1\,. \tag{6.5}$$

This regime, as mentioned earlier, is non-inflationary because the fluxes are effectively vanishing and therefore the membrane (or the D3 brane) is almost stationary (reftempom).[46] We need not restrict ourselves to this regime, and can study the issue of semi-local defects in the full inflationary set-up as was done earlier in [30]. The point however is that, as we increase the global symmetries, the number of scalars increases and therefore usual inflationary dynamics may not be easy to realise here. For simplicity, we will restrict ourselves to a region on or near (6.5). The warp factor $B(y)$ now has the following representation:

$$B(y) = b(y) - \frac{1}{12}\sum_{i=1}^{n} \log(z - z_i) + \text{c.c} + \text{modular functions} \tag{6.6}$$

where $b(y)$ would be some as yet undetermined function of the internal space, and we have put in modular functions to have a consistent description of warp factors on a torus.

[46] It is also interesting to note that in this region (6.5) the two backgrounds that we studied i.e the *deformed* conifold of section 3.5 and the $K3 \times \mathbb{P}^1$ background of [25] would look effectively similar, and therefore would have an approximate $\mathcal{N} = 2$ supersymmetry. Globally the two backgrounds are quite different of course as we saw earlier.

Finally, the integer n determines the number of seven branes, with z_i being the position of seven branes on the (x, θ_1) torus.

The z_i behavior of the warp factor B will not change even when we switch on non-primitive fluxes, although the function $b(y)$ might get more complicated. Thus going to a slightly different regime of our model, we are able to infer the y^m dependences of the warp factor B. The other warp factor $A(y)$ still remains undetermined (at least using this technique), although if there is some special relation between A and B, we might expect to get some information about A also.

6.1 Examples of semilocal defects

Having set-up the stage to study semilocal defects, we should now venture towards concrete examples. The semilocal defects are determined with respect to global as well as local symmetries. The global symmetries are related to the singularities of our manifold [144–146]. Various singularity types would determine what global symmetries we should expect in this set-up.

The local symmetries are determined by the number of D3 branes (or M2 branes in M-theory) that we would put in. Once we determine both the local and global symmetries, we look for energetics of the system to see whether we should expect semilocal defects or not. The energetics can sometime be specified by the ratio of Higgs mass m, and vector Boson mass M as $\beta = \frac{m^2}{M^2}$ from the D3 brane point of view. For example, consider a case where the Higgs boson is much heavier than the vector boson. Having a massive vector means that we expect magnetic fluxes to confine to a finite region. However as shown in [52] and [140] this doesn't quite happen because there are no stable vortices. Thus energetics plays an important role here, and that is one of the key reasons why these defects are termed non-topological.

Coming back to the issue of global symmetries, it is by now well known that ADE type of singularities can easily arise here once we have the following situation:

$$\begin{aligned}
\mathbf{A_n} &: \; z^{n+1}, \qquad n \geq 1, \\
\mathbf{D_n} &: \; x^{n-1} + xz^2, \qquad n \geq 4 \\
\mathbf{E_6} &: \; x^3 + z^4, \\
\mathbf{E_7} &: \; x^3 + xz^3, \\
\mathbf{E_8} &: \; x^3 + z^5
\end{aligned} \tag{6.7}$$

where x, z would be related to the coordinates of the T^2 fiber or/and to the (x, θ_1) space in a suitable way.

To determine the relation between the Weierstrass equation that we presented earlier and the singularity types above, (6.7), we have to write an equation that is more generic than the Weierstrass equation [145], namely:

$$y^2 + a_1 xy + a_3 y = x^3 + a_2 x^2 + a_4 x + a_6 \tag{6.8}$$

where a_i are in general functions of the base coordinate z, and if we replace x in the above equation (6.8) by $x - \frac{b_2}{12}$, then (6.8) can take the following form:

$$\left(y + \frac{12a_1x - a_1b_2 + 12a_3}{24}\right)^2 = x^3 - \frac{x}{48}\left(a_1^4 + 16a_2^2 + 8a_1^2a_2 - 24a_1a_3 - 48a_4\right) + \\ + \frac{1}{864}\Big(a_1^8 + 12a_1^4a_2 + 48a_1^2a_2^2 + 64a_2^3 + 216a_3^2 - 36a_1^3a_3 - \\ - 72a_1^2a_4 - 144a_1a_2a_3 - 288a_2a_4 + 864a_6\Big) \quad (6.9)$$

with $b_2 = 4a_2 + a_1^2$. In this form it is now easy to identify the Weierstrass equation that we presented earlier; and once we have the Weierstrass equation it is a simple exercise to extract the singularity types that we want for our case. In our earlier paper [30] we gave a possible F-theory configuration that is responsible for exceptional global symmetries in the $K3 \times \mathbb{P}^1$ background. These configurations have been discussed earlier in [147]. The F-theory curves can be extracted from there. What we now require is an analysis similar to the one done by [56] for the existence of semilocal strings in our model with higher global symmetries.

The SU$(n+1)$ global symmetry follows in a straightforward manner. We have already given a concrete way to realize SU(2) global symmetry in [30]. What we need is simply to fix the discriminant behavior as

$$\Delta \sim z^{n+1} + \mathcal{O}(z^{n+2}) \quad (6.10)$$

upto an overall numerical factor. The $A_1, A_2, A_3..$ cases would therefore go as $z^2, z^3, z^4..$ respectively. A more detailed representation of the singularities can also be obtained when the T^2 fiber degenerates over a four-dimensional subspace of the base. This is when we would need (z_1, z_2) coordinates to specify f, g in the Weierstrass equation. For our case we have to fix $f(z)$ and $g(z)$ in such a way as to realize (6.10) correctly. The local symmetry is just U(1) coming from the D3 brane that is fixed far away. The breaking pattern here will be:

$$\frac{\mathrm{SU}(n+1)_g \times \mathrm{U}(1)_l}{\mathbb{Z}_{n+1}} \quad (n > 1) \quad \xrightarrow{\Phi} \quad \mathrm{U}(n)_g\,, \quad (6.11)$$

where the subscript g, l refer to global and local respectively. The vacuum manifold is S^{2n+1}, which basically comes from the coset space of our groups; and since $\pi_1\left(\frac{\mathrm{U}(n+1)}{\mathrm{U}(n)}\right) = 1$, there could only be semi-local strings. Once the energetics favor them, the Higgs fields wrap the S^1 directions of the Hopf fibration [56]:

$$S^{2n+1} \quad \xrightarrow{S^1} \quad \mathbb{CP}^n\,. \quad (6.12)$$

These S^1 are related to the local U(1) gauge symmetry that the single D3 brane has on its world volume. Thus there would be infinite set of strings with Higgs fields wrapping various S^1 that cannot be deformed into one another with *finite* energies.

The U(1) local symmetry that we discussed for the D3 brane is only residual. The actual local symmetry would be Sp(1) broken to U(1) at all points in moduli space [148]. In fact for m D3 branes we expect to see an unbroken gauge group of Sp(m) [149] (where only a subgroup of this will be realized in the Coulomb branch of the corresponding $\mathcal{N} = 2$ gauge theory[47]). For the semi-local defects, then we would be interested in studying global symmetries that are of the form $\mathrm{Sp}(n+1)$.

The $\mathrm{Sp}(n+1)$ symmetries are not that easy to realise in a conventional way from Tate's algorithm (see discussion in [145]). We will therefore give an approximate way to see these from (6.9), and then try to realize them directly from the D3 brane world volume using the construction of [59, 60, 150]. The Weierstrass equation from (6.8) for this case can be written as

$$y^2 - x^3 + a_1 xy + a_2 x^2 + a_3 y z^{n+1} + a_4 x z^{n+1} + a_6 z^{2n+2} = 0\,, \tag{6.13}$$

where a_i are in general arbitrary, and the behavior of the discriminant for the case when we convert (6.13) to the standard Weierstrass form (6.9), will tell us that

$$\Delta \sim z^{2n+2} + \mathcal{O}(z^{2n+3}) \tag{6.14}$$

up to an overall numerical factor. One can verify that for $\mathrm{Sp}(1) \approx \mathrm{SU}(2)$ the leading order behavior of the discriminant is z^2, which is what we got earlier. The above analysis therefore gives us global $\mathrm{Sp}(n+1)$ symmetries, up to possible subtleties mentioned in [145]. Therefore for a D3 brane near the seven branes we expect to see the following breaking:

$$\frac{\mathrm{Sp}(n+1)_g \times \mathrm{Sp}(1)_l}{\mathbb{Z}_2} \quad \xrightarrow{\Phi} \quad \frac{\mathrm{Sp}(n)_g \times \mathrm{SU}(2)_g}{\mathbb{Z}_2}\,, \tag{6.15}$$

which is exactly what was discussed in [56]. Our case can however be even more general that the one presented in [56] and mentioned above, because we can put m D3 branes to get an additional local symmetries of $\mathrm{Sp}(m)$.

The vacuum manifold is a $4n+3$ dimensional sphere S^{4n+3}. What about gauge orbits? The local gauge symmetry is $\mathrm{Sp}(m)$, and for D3 branes one would expect (at least classically) a gauge orbit for $\mathrm{SU}(2)$ group, which is of course the three sphere S^3. Thus the equivalent picture of (6.12) for this case would be [56]

$$S^{4n+3} \quad \xrightarrow{S^3} \quad \mathbb{HP}^n \tag{6.16}$$

where $\mathbb{HP}^n$ is the *quaternionic* projective space (recall that earlier we had complex projective space (6.12)).

From our D3 brane(s) picture, such a vacuum manifold is not difficult to find. The $(n+1)$-plet complex scalar fields (i.e the strings to the different seven branes) can be theoretically replaced by $(n+1)$ quaternions. The space that we get from doing this is almost equivalent to a Kähler manifold in the sense that there may sometime exist a globally defined complex structure $J_j{}^i$ which is written in terms of three locally defined (1,1) tensors as

$$J_j{}^i = \alpha_{\mathrm{x}} J_j^{\mathrm{x}\ i} + \alpha_{\mathrm{y}} J_j^{\mathrm{y}\ i} + \alpha_{\mathrm{z}} J_j^{\mathrm{z}\ i} \tag{6.17}$$

[47]Recall that we are in the subspace (6.5) where we may see an approximate $\mathcal{N} = 2$ theory on the D3 brane probes.

where $\alpha_{\mathrm{x,y,z}}$ are constants. The complex structure $J_j{}^i$ may or may not be covariantly constant. When it is covariantly constant then it is Kähler of course. On the other hand, the tensors $J_j^{\mathrm{x}\ i}, J_j^{\mathrm{y}\ i}, J_j^{\mathrm{z}\ i}$ satisfy a quaternion algebra

$$J_i^{\mathrm{x}\ j} J_j^{\mathrm{y}\ k} = -\delta^{\mathrm{xy}} \delta_i{}^k + \epsilon^{\mathrm{xyz}} J_i^{\mathrm{z}\ k} \,. \tag{6.18}$$

A quick consistency check of the above analysis would be to ask whether restoration of supersymmetry would allow such vacuum manifold. From the analysis that we presented above, vanishing flux condition would lead to an approximate $\mathcal{N} = 2$ supergravity This is perfectly consistent with our picture as pointed out long ago by Bagger-Witten [151] that local $\mathcal{N} = 2$ supersymmetry requires the vacuum manifold to be a quaternionic Kähler manifold with negative scalar curvature that is proportional to the Newton's constant.[48] In the limit of vanishing scalar curvature we expect to get a hyper-Kähler manifold as our vacuum manifold. This is of course a well known story [152].

For the example that we presented here in (6.16), the simplest case would be when the base manifold is $\mathbb{HP}^1$. This is a four-sphere S^4 which is a quaternionic Kähler manifold *without* a complex structure. There are also other quaternionic spaces that are not of the form $\mathbb{HP}^n$. In fact as shown by [59] and [60], there are two more varieties given by

$$X\mathbb{P}^n = \frac{\mathrm{SU}(n+2)}{\mathrm{U}(n) \otimes \mathrm{U}(2)} \,, \qquad Y\mathbb{P}^n = \frac{\mathrm{SO}(n+4)}{\mathrm{SO}(n) \times \mathrm{SO}(4)} \,, \tag{6.19}$$

where $\otimes$ is used to denote the product $\mathrm{U}(n) \times \mathrm{U}(2)$ with unit determinant.

Comparing $X\mathbb{P}^n$ with our earlier analysis, we see that once we know the singularity equation for the A_n algebra it is not difficult to realise this coset. When $n = 1$, $X\mathbb{P}^1 = \mathbb{CP}^2$, which we have already determined. Therefore it only remains to determine the Weierstrass equation for $\mathrm{SO}(n+4)$, then the second coset can also be realised. This equation is rather easy to extract from Tate's algorithm so we will not do it here. Interested readers may want to work it out by themselves. We simply point out an obvious group equality

$$Y\mathbb{P}^1 = \frac{\mathrm{SO}(5)}{\mathrm{SO}(4)} = \frac{\mathrm{Sp}(2)}{\mathrm{Sp}(1) \times \mathrm{Sp}(1)} = S^4 = \mathbb{HP}^1 \,, \tag{6.20}$$

which is again consistent from our vacuum manifold construction. Thus this way all the defects associated with A_n, B_n, C_n and D_n global symmetries can be realised in our model. What remains now are the exceptional ones.

To get the semilocal defects associated with exceptional global symmetries, we have to back up a little for more generality. In the process we will also be able to shed some light on the classification of *homogeneous* quaternionic Kähler manifold done many years ago by Wolf [59] and Alekseevski [60] and generalized by [61]. The key idea governing the formation of semilocal defects can be rephrased in the following way:

- Given any global group with the corresponding algebra $\mathcal{G}$, first find the *maximal* regular subalgebra $\mathcal{H}$. Having a maximal subalgebra would mean that we can ignore the $u(1)$ groups.

[48] On the other hand, $\mathcal{N} = 1$ supersymmetry requires the vacuum manifold to be a Kähler one [151].

- The subalgebra should be expressible in terms of product of two smaller subalgebras, namely $\mathcal{H} = \mathcal{H}_1 \times \mathcal{H}_2$.

- One of $\mathcal{H}_i$ should form the *local* gauge symmetry on the D3 brane(s). For example let us assume $\mathcal{H}_1$ to be the allowed local algebra (or the local group) on the D3 brane(s).

- The local group[49] $\mathcal{H}_1$ should have the required homotopy classification $\pi_n(\mathcal{H}_1^c) = \mathbb{Z}$, where $\mathcal{H}_1^c$ is the coset for the group $\mathcal{H}_1$.

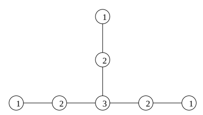
Then semilocal defects can form on the D3 brane(s) world volume with a global symmetry $\mathcal{G}$, provided the energetics also allows. The generic breaking pattern of the groups in this case will be:

$$\mathcal{G}_g \times (\mathcal{H}_1)_l \quad \xrightarrow{\Phi} \quad (\mathcal{H}_2)_g \times (\mathcal{H}_1)_g \tag{6.21}$$

where the subscript g and l refer to global and local symmetries respectively, as before. The corresponding coset manifold $\mathcal{M}_G$, for which every point p corresponds to a semilocal defect that cannot be deformed into another one with finite energy, is given by

$$\mathcal{M}_G = \frac{\mathcal{G}}{\mathcal{H}_1 \times \mathcal{H}_2}\,. \tag{6.22}$$

Many of the cases that we studied so far (or have been addressed in the literature) can be seen to follow from the above framework. Therefore let us first use this to study the case when $\mathcal{G} = E_6$. The maximal regular subalgebra of E_6 can be extracted from the extended Dynkin diagram



and is given by $\mathcal{H} = su(6) \times su(2)$. This immediately tells us two things: One, the world volume gauge group $\mathcal{H}_1 = \mathrm{SU}(2) = \mathrm{Sp}(1)$, and two, the manifold $\mathcal{M}_{E_6}$ is

$$\mathcal{M}_{E_6} = \frac{E_6}{\mathrm{SU}(6) \times \mathrm{Sp}(1)}\,. \tag{6.23}$$

Since $\mathrm{SU}(2) \sim S^3$, the homotopy classification will tell us that $\pi_3(S^3) = \mathbb{Z}$. These are the instantons, and therefore should have a construction via the quaternion as we discussed before. Having a quarternionic framework also means that $\mathcal{M}_{E_6}$ should be a quaternionic Kähler manifold if whatever we said is consistent. One can easily check from [59, 60] that this is exactly the case!

Therefore we seem to get our first exceptional semilocal defect in this model. However in the process of deriving this we have ignored a subtlety. This subtlety cannot be seen at the level of group structure, but is visible when we look at the F-theory curve associated

[49] We use the same notation for the group and its algebra. The distinction between them doesn't affect the analysis below.

with our manifold. Therefore let us construct the corresponding F-theory curve. From Tate's algorithm this is given by

$$y^2 - x^3 - a_1 z^5 + a_2 xyz - a_3 x^2 z^2 - a_4 x z^3 + a_5 y z^2 = 0 \tag{6.24}$$

with a_i arbitrary. Once we convert this equation to the familiar form of (6.9) the discriminant locus can be easily worked out. For us this will be given by

$$\Delta \sim z^8 + \mathcal{O}(z^9) \tag{6.25}$$

up to an overall numerical factor. Knowing the discriminant we can in principle extract the corresponding subalgebra associated with the global group $\mathcal{G} = E_6$ provided the background space is specified. We have already chosen our subspace that allowed us earlier to realise an approximate $\mathcal{N} = 2$ supersymmetry. This is of course the space given by the equation (6.5). The F-theory curve (6.24) with discriminant (6.25) will then give us the following subalgebra on this space:

$$su(5) \times su(2) \times u(1) \tag{6.26}$$

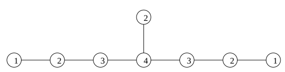
which is almost the maximal subalgebra that we wanted, but not quite. In fact $su(6)$ is broken to $su(5) \times u(1)$. Thus this is the closest we come to getting the full structure of the coset space directly from type IIB string theory (or F-theory). How do we then realize the full configuration? First we decompose the E_6 adjoint in terms of the subalgebra (6.26) as

$$\mathbf{78} = (\mathbf{24},\mathbf{1})_0 + (\mathbf{1},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 + (\mathbf{10},\mathbf{2})_{-3} + (\mathbf{5},\mathbf{1})_6 + \text{c.c} \tag{6.27}$$

where the subscripts refer to the U(1) charges and the c.c are associated with $\bar{10}$ and $\bar{5}$ with U(1) charges 3 and -6 respectively.

Secondly, having given the decomposition, the rest of the discussion now should follow the familiar line developed in the series of papers [147, 153]. We will not elaborate on this aspect as the readers can look up the details in those papers. It'll simply suffice to mention that the non-trivial configuration required to get the full group structure lies in the process of brane creation via the Hanany-Witten effect [142] leading to strings with multiple prongs [154 – 156] that fill out the rest of the group generators [153].

The above construction therefore gives us the semilocal instanton configurations associated with global symmetry E_6. Let us now turn towards the next group $\mathcal{G} = E_7$. The extended Dynkin diagram of E_7



can be *cut* in different ways to give rise to various maximal regular subalgebras of E_7. They are given by

$$su(8)\,, \qquad \text{spin}(12) \times su(2)\,, \qquad su(6) \times su(3)\,, \qquad su(4) \times su(4) \times su(2) \tag{6.28}$$

where spin(12) actually comes from $so(12)$ with some identification between the generators. From the set of steps that we mentioned earlier, we can immediately ignore $su(8)$ and

$su(4) \times su(4) \times su(2)$ as they are not product of two subalgebras. What about the other two cases? To see which one we could keep, we have to look for the *homotopy* classification of the local gauge group on the D3 brane(s). Taking $\mathcal{H}_1$ as either $su(2)$ or $su(3)$ we see that[50]

$$\pi_3\left(su(2)\right) = \pi_3\left(su(3)\right) = \mathbb{Z}\,. \tag{6.29}$$

Thus homotopically both the groups can exist. But since on a single D3 brane we can either have U(1) (quantum mechanically) or Sp(1) (classically), the case with $\mathcal{H}_1 = \mathrm{SU}(3)$ cannot arise here. The above consideration immediately gives us the corresponding unique coset manifold for the global symmetry E_7 as

$$\mathcal{M}_{E_7} = \frac{E_7}{\mathrm{Spin}(12) \times \mathrm{Sp}(1)}\,. \tag{6.30}$$

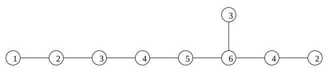
Our previous consideration will require us to view this as a homogeneous quaternionic Kähler manifold. From the classification of [59, 60] we see that this is indeed the case.

Here again we face the same kind of subtlety, as for the E_6 case, when we look into the F-theory curve. We will not go into the details of this as the curve and its behavior in the sub-space (6.5) can be easily worked out. The discriminant locus and the corresponding subalgebra that we can realise here will be, respectively,

$$\Delta \sim z^9 + \mathcal{O}(z^{10})\,, \qquad su(6) \times su(2) \times u(1) \tag{6.31}$$

and therefore once we decompose **133** over this subalgebra we can use multi-prong strings to realize the full group structure [153].

The final exceptional global symmetry that we want to study here is E_8. This is straightforward. The extended Dynkin diagram is now given by



From here the relevant allowed maximal subalgebra is $E_7 \times su(2)$. Other choices can be discarded because of the absence of $su(2)$ factor or non-decomposability as products of two smaller subalgebras. Thus this will give us the next coset manifold:

$$\mathcal{M}_{E_8} = \frac{E_8}{E_7 \times \mathrm{Sp}(1)}\,. \tag{6.32}$$

From the construction of [59, 60] the readers can check that we have realized the final homogeneous quaternionic Kähler manifold. Next looking at the discriminant on the sub-space (6.5) we get the manifest subalgebra: $su(7) \times su(2) \times u(1)$, on which we could decompose the adjoint **248** and recover the full structure via [153]. This way we get a "physical" reason for the existence of the homogeneous quaternionic Kähler manifolds. Namely: they are related to the existence of semilocal defects on a D3 brane!

[50]In fact generically $\pi_3\left(su(n)\right)|_{n\geq 2} = \mathbb{Z}$. Similarly $\pi_3\left(so(n)\right)|_{n\geq 3, n\neq 4} = \mathbb{Z}$ and $\pi_3\left(so(4)\right) = \pi_3\left(su(2)\times su(2)\right) = \mathbb{Z}\oplus\mathbb{Z}$. For Exceptional groups $\pi_m\left(E_n\right) = \mathbb{Z}\delta_{m3}$. For more details see [157].

Before finishing, we should point out two things:

- First, instead of the choice of *maximal* subalgebras, we could have also asked for *symmetric* subalgebras of the groups. The symmetric subalgebras for various groups have been listed in [158]. For the A_n, B_n, C_n, D_n cases, they are

$$\begin{aligned} su(p+q) &\to su(p) \times su(q) \times u(1)\,, \\ so(p+q) &\to so(p) \times so(q) \\ sp(2p+2q) &\to sp(2p) \times sp(2q) \end{aligned} \tag{6.33}$$

  where p and q form the various distribution (as even or odd integers). For the E_n cases one would have

$$\begin{aligned} &E_8 \to so(16)\,, \qquad su(2) \times E_7 \\ &E_7 \to su(8)\,, \qquad su(2) \times so(12)\,, \qquad E_6 \times u(1) \\ &E_6 \to sp(8)\,, \qquad su(2) \times su(6)\,, \qquad so(10) \times u(1)\,, \qquad F_4\,. \end{aligned} \tag{6.34}$$

  From the list one has to extract out the relevant algebras that we would require for our case.

- Second, there are two more coset spaces given in terms of G_2 and F_4 algebra as

$$\frac{G_2}{\mathrm{SU}(2) \times \mathrm{Sp}(1)}, \qquad \text{and} \qquad \frac{F_4}{\mathrm{Sp}(3) \times \mathrm{Sp}(1)}\,. \tag{6.35}$$

  The structure of these follow exactly the criteria that we laid above (notice the Sp(1) factor).[51] The F-theory curves for these cases can be worked out from Tate's algorithm. But a full analysis using multi-prong strings [154 − 156] *à la* [153] has not been done. So we will leave this for the sequel to this paper.

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[51] The symmetric subalgebras for the group F_4 are $sp(1) \times sp(3)$ and $so(9)$; and for G_2 is $su(2) \times sp(1)$.

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